SSLC Mathematics.

Content 2023-24

Lesson-1.

Arithmetic Progressions.

Definition of AP :

A progression in which the difference between any two consecutive terms is a constant is called an arithmetic progression (AP).

General Form of AP :

If 'a' is the first term and 'd' is the common difference of an AP, then the general form of the AP is,

$a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots$

General term of AP :

If 'a' is the first term and 'd' is the common difference of an AP, then the general term of the AP is,

$a_n = a + (n-1)d$

Arithmetic series :

The sum of the terms of a progression is called a series.

The sum of the terms of an arithmetic progression is called arithmetic series.

The general form of an arithmetic series is,

$a + (a + d) + (a + 2d) + \ldots\ldots$

Sum of first 'n' natural numbers :

Sum of first 'n' natural numbers is S_n and is given by,

$S_n = 1 + 2 + 3 + 4 + \ldots\ldots\ldots + n.$

$$S_n = \frac{n(n+1)}{2}$$

Sum of first 'n' terms of an AP :

The sum of first 'n' terms of an AP is denoted by S_n and is given by,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

If the last term of the AP is given, then

$$S_n = \frac{n}{2}(a + l)$$

1) Which term of the AP 3, 8, 13, ... is 73?

Solution :

3, 8, 13, ... 73

$a_n = a + (n-1)d$	$a = 3.$
$73 = 3 + (n-1)5$	$d = 8 - 3.$
$73 - 3 = (n-1)5$	$= 5$
$70 = 5(n-1)$	$a_n = 73$
$n - 1 = \frac{70}{5}$	$n = ?$

$n - 1 = 14$

$n = 14 + 1$

$n = 15$

2) How many two-digit numbers are divisible by 3?

Solution :

12, 15, 18, ... 99.

$a_n = a + (n-1)d$	$a = 12$
$99 = 12 + (n-1)3$	$d = 15 - 12$
$99 - 12 = 3(n-1)$	$= 3$
$87 = 3(n-1)$	$a_n = 73$
$(n - 1) = \frac{87}{3}$	$n = ?$

$n - 1 = 29$

$n = 29 + 1$

$n = 30$

3) Find the number of terms in the AP 100, 96, 92, ... 12.

Solution :

100, 96, 92, ... 12	$a = 100$
$a_n = a + (n-1)d$	$d = 96 - 100$
$12 = 100 + (n-1)(-4)$	$= -4$
$12 - 100 = (n-1)(-4)$	$a_n = 12$
$-88 = (n-1)(-4)$	$n = ?$

$n-1=\frac{-88}{-4}$

$n-1=22$

$n=22+1$

$n=23$

4) Find the number of terms in the arithmetic progression 7, 13, 19, ... 205.

5) Find the sum of first 20 terms of an AP 3, 7, 11, 15, ...

Solution :

$3, 7, 11, 15, \ldots\ldots.\ 20\ terms$ | $a = 3$, $d = 7-3 = 4$, $n = 20$, $Sn = ?$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{20}=\frac{20}{2}\ [2(3)+(20-1)4]$

$= 10\ [\ 6\ +\ (19)4]$

$= 10\ (6\ +\ 76)$

$= 10\ (82)$

$= 820.$

6) Find the sum of first 23 terms of the AP 2, 7, 12, ...

Solution :

$2, 7, 12, \ldots\ldots.\ upto\ 23\ terms$ | $a = 2$, $d = 7-2 = 5$, $n = 23$, $Sn = ?$

$S_{23}=\frac{23}{2}[2(2)+(23-1)5]$

$=\frac{23}{2}[4+(22)5]$

$=\frac{23}{2}[4+110]$

$=\frac{23}{2}[114]$

$= 23\ \times 57$

$= 1311$

7) Find the sum of first 20 terms of the series $5+10+15+\ \ldots\ldots.$

8) Find the sum of first 10 terms of the series $5+8+11+\ \ldots\ldots.$

9) Find the sum of all the multiples of 4 lying between 10 and 250.

Solution :

$12+16+20+\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ +248$

$a = 12$, $d = 16-12 = 4$, $a_n = 248$, $n = ?$

$a_n=a+(n-1)d$

$248=12+(n-1)4$

$248-12=(n-1)4$

$236=(n-1)4$

$n-1=\frac{236}{4}$

$n-1=59$

$n=59+1$

$n=60$

$S_n=\frac{n}{2}(a+l)$

$S_{60}\ =\frac{n}{2}(12+248)$

$=\frac{60}{2}(260)$

$=30\times 260$

$=780$

10) Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solution :

$a_3\ =\ 5.$

$a\ +\ 2d\ =\ 5\ldots\ldots\ldots\ldots..(1)$

$a_7\ =\ 9$

$a\ +\ 6d\ =\ 9\ldots\ \ldots\ldots\ldots.(2)$

$(2)-(1) gives$

$a\ +\ 6d\ =\ 9$

$a\ +\ 2d\ =\ 5$

$4d\ =\ 4$

$d=\frac{4}{4}$

$d\ =\ 1.$

$Substitute\ in\ (1)$

$a + 2d = 5$

$a + 2(1) = 5$

$a + 2 = 5$

$a = 5 - 2$

$a = 3$

$\therefore AP\ is\ 3, 4, 5, 6, \ldots \ldots \ldots \ldots$

11)An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

12)Determine the AP whose third term is 16 and the seventh term exceeds the fifth term by 12.

Solution :

$a_3 = 16$

$a + 2d = 16 \ldots\ldots\ldots\ldots\ldots (1)$

$a_7 = a_5 + 12$

$a + 6d = a + 4d + 12.$

$6d - 4d = 12$

$2d = 12$

$d = \frac{12}{2}$

$d = 6.$

$Substitute\ in\ (1)$

$a + 2d = 16$

$a + 2(6) = 16$

$a + 12 = 16$

$a = 16 - 12$

$a = 4.$

$AP\ is\ 4, 10, 16, 22, \ldots\ldots\ldots\ldots\ldots$

13)The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the AP.

Solution

$a_4 + a_8 = 24$

$a + 3d + a + 7d = 24.$

$2a + 10d = 24$

$Divide\ by\ 2,$

$a + 5d = 12 \ldots\ldots\ldots\ldots\ldots (1)$

$a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44.$

$2a + 14d = 44$

$Divide\ by\ 2,$

$a + 7d = 22 \ldots\ldots\ldots\ldots\ldots (2)$

$(2) - (1)\ gives$

$a + 7d = 22$

$a + 5d = 12$

$2d = 10$

$d = \frac{10}{2}$

$d = 5$

$Substitute\ in\ (1)$

$a + 5d = 12$

$a + 5(5) = 12$

$a + 25 = 12$

$a = 12 - 25.$

$a = -13.$

$AP\ is\ -13, -8, -3, 2, 7, \ldots\ldots\ldots\ldots$

14)Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

15)In an AP, if the 12th term is -13 and the sum of its first four terms is 24, find the sum of its first 20 terms.

Solution :

$a_{12} = -13$

$a + 11d = -13 \ldots\ldots\ldots\ldots\ldots (1)$

$a_1 + a_2 + a_3 + a_4 = 24$

$a + a + d + a + 2d + a + 3d = 24$

$4a + 6d = 24 \ldots\ldots\ldots\ldots\ldots (2)$

$(1) \times 4 - (2) gives$

$4a + 44d = -52$

$4a + 6d = 24$

$38d = -76$

$d = -\frac{76}{38}$

$d = -2$

$Substitute\ in\ (1)$

$a + 11d = -13$

$a + 11(-2) = 14$

$a - 22 = -13$

$a = -13 + 22$

$a = 9$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2(9) + (20-1)(-2)]$

$= 10[18 + (19)(-2)]$

$= 10[18 - 38]$

$= 10(-20)$

$= -200$

16)The 14th term of an AP is twice the 8th term. If the 6th term is -8, then find the sum of its first 20 terms.

Solution :

$a_{14} = 2 \times a_8$

$a + 13d = 2(a + 7d)$

$a + 13d = 2a + 14d$

$a - 2a = 14d - 13d$

$-a = d$

$a = -d$

$a_6 = -8$

$a + 5d = -8$

$a + 5(-a) = -8$

$a - 5a = -8$

$-4a = -8$

$a = \frac{8}{4}$

$a = 2$

$\therefore\ d = -2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2(2) + (20-1)(-2)]$

$= 10[4 + (19)(-2)]$

$= 10[4 - 38]$

$= 10(-24)$

$= -240$

17)If the sum of 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution :

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2a + (7-1)d]$

$49 = \frac{7}{2}[2a + 6d]$

$49 = \frac{7}{2}[2(a + 3d)]$

$49 = 7\,(a + 3d)$

$a + 3d = \frac{49}{7}$

$a + 3d = 7 \ldots\ldots\ldots\ldots\ldots (1)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{17} = \frac{17}{2}[2a + (17-1)d]$

$289 = \frac{17}{2}[2a + 16d]$

$289 = \frac{17}{2}[2(a + 8d]$

$289 = 17(a + 8d]$

$a + 8d = \frac{289}{17}$

$a + 8d = 17 \ldots\ldots\ldots\ldots\ldots\ldots (2)$

$(2) - (1) gives$

$a + 8d = 17$

$a + 3d = 7$

$5d = 10$

$d = \frac{10}{5}$

$d = 2$

$Substitute\ in\ (1)$

$a + 3d = 7$

$a + 3(2) = 7$

$a + 6 = 7$

$a = 7 - 6$

$a = 1$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2(1) + (n - 1)2]$

$= \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2}[2n]$

$= n^2$

18)Find the sum of the first 15 terms of an AP whose nth term is, $a_n = 9 - 5n$.

$a_n = 9 - 5n$

$a_1 = 9 - 5(1)$

$= 9 - 5$

$= 4$

$a_n = 9 - 5n$

$a_2 = 9 - 5(2)$

$= 9 - 10$

$= -1$

$d = a_2 - a_1$

$= -1 - 4$

$= -5$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{15} = \frac{15}{2}[2(4) + (15 - 1)(-5)]$

$= \frac{15}{2}[8 + (14)(-5)]$

$= \frac{15}{2}[14 - 70]$

$= \frac{15}{2} \times 56$

$= 15 \times 28$

$= 420$

19)Find the sum of the first 15 terms of an AP whose nth term is, $a_n = 3 + 4n$.

20)If the sum of first n terms of an AP is $4n - n^2$. Find the 10th term.

<u>Solution</u> :

$S_n = 4n - n^2$

$S_1 = 4(1) - (1)^2$

$= 4 - 1$

$= 3$

$a = 3$

$S_2 = 4(2) - (2)^2$

$a_1 + a_2 = 8 - 4$

$3 + a_2 = 4$

$a_2 = 4 - 3.$

$a_2 = 1$

$d = a_2 - a_1$

$= 1 - 3$

$= -2$

$a_n = a + (n-1)d$

$a_{10} = a + (10-1)d$

$= 3 + 9(-2)$

$= 3 - 18$

$= -15$

21)If the sum of first 7 terms of an AP is 182 and 4th term and 17th terms are in the ratio 1 : 5, then find the AP.

Solution :

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2a + (7-1)d]$

$182 = \frac{7}{2}[2a + 6d]$

$182 = \frac{7}{2} \times 2(a + 3d)$

182 = 7 (a + 3d)

$a + 3d = \frac{182}{7}$

$a + 3d = 26 \ldots\ldots\ldots\ldots (1)$

$\frac{a_4}{a_{17}} = \frac{1}{5}$

$\frac{a + 3d}{a + 16d} = \frac{1}{5}$

$(a + 3d)5 = a + 16d$

$5a + 15d = a + 16d$

$5a - a = 16d - 15d$

$4a = d$

$Or\ d = 4a$

Substitute in (1)

$a + 3(4a) = 26$

$a + 12a = 26$

$13a = 26$

$a = \frac{26}{13}$

$a = 2$

$\therefore d = 4(2)$

$d = 8$

$\therefore AP\ is\ 2, 10, 18, \ldots\ldots\ldots$

22)The first term of two AP's are equal and the ratios of their common differences is 1 : 2. If the 7th term of first AP and 21st term of second AP are 23 and 125 respectively. Find two AP's.

Solution :

$a_7 = 23$

$a + 6d = 23 \ldots\ldots\ldots\ldots\ldots\ldots (1)$

$a_{21} = 125$

$a + 20(2d) = 125$

$a + 40d = 125 \ldots\ldots\ldots\ldots\ldots (2)$

$(2) - (1)\ gives$

$a + 40d = 125$

$a + 6d = 23$

$34d = 102$

$d = \frac{102}{34}$

$d = 3$

$Substitute\ in\ (1)$

$a + 6d = 23$

$a + 6(3) = 23$

$a + 18 = 23$

$a = 23 - 18$

$a = 5$

$1st\ AP\ :\ a = 5, d = 3$

$5, 8, 11, 14, \ldots\ldots\ldots\ldots$

$2nd\ AP\ :\ a = 5, d = 6$

$5, 11, 17, 23, \ldots\ldots\ldots\ldots$

23)In an AP whose first term is 2, the sum of first five terms is one fourth the sum of the next five terms. Show that $a_{20} = -112$ and find S_{20}.

Solution :

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_5 = \frac{5}{2}[2a + (5-1)d]$

$= \frac{5}{2}[2a + 4d]$

$= \frac{5}{2} \times 2(a + 2d)$

$= 5(a + 2d)$

$= 5a + 10d$

$= 5(2) + 10d$

$= 10 + 10d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + (10-1)d]$

$= 5(2a + 9d)$

$= 10a + 45d$

$= 10(2) + 45d$

$= 20 + 45d$

$S_5 = \frac{1}{4}(5_{10} - S_5)$

$4(10 + 10d) = (20 + 45d - 10 - 10d)$

$40 + 40d = 10 + 35d$

$40d - 35d = 10 - 40$

$5d = -30$

$d = -\frac{30}{5}$

$d = -6$

$a_n = a + (n-1)d$

$a_{20} = a + (20-1)(-6)$

$= 2 + 19(-6)$

$= 2 - 114$

$= -112$

$S_n = \frac{n}{2}[a + l]$

$S_{20} = \frac{20}{2}[2 + a_{20})]$

$= 10[2 + (-112)]$

$= 10[2 - 112]$

$= 10(-110)$

$= -1100$

24)In an arithmetic progression, the sum of first 11 terms is 44 and the sum of next 11 terms is 55, find the first term and common difference.

Solution :

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{11} = \frac{11}{2}[2a + (11-1)d]$

$44 = \frac{11}{2}[2a + 10d]$

$88 = 11(2a + 10d)$

$2a + 10d = \frac{88}{11}$

$2a + 10d = 8$ ....(1)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{22} = \frac{22}{2}[2a + (22-1)d]$

$55 + 44 = 11[2a + 21d]$

$99 = 11(2a + 21d)$

$2a + 21d = \frac{99}{11}$

$2a + 21d = 9$............... (2)

$(2) - (1)$ *gives*

$2a + 21d = 9$

$2a + 10d = 8$

$11d = 1$

$d = \frac{1}{11}$

25)Three numbers are in arithmetic progression and their sum is 18 and the sum of their squares is 140. Find the numbers.

Solution :

$Let\ the\ numbers\ be\ (a - d), a\ and\ (a + d)$

$Sum = 18$

$a - d + a + a + d = 18$

$3a = 18$

$a = \frac{18}{3}$

$a = 6.$

$Sum\ of\ the\ squares = 140$

$(a - d)^2 + a^2 + (a + d)^2 = 140$

$a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 140$

$3a^2 + 2d^2 = 140$

$3(6^2) + 2d^2 = 140$

$3(36) + 2d^2 = 140$

$108 + 2d^2 = 140$

$2d^2 = 140 - 108$

$2d^2 = 32$

$d^2 = \frac{32}{2}$

$d^2 = 16$

$d = \sqrt{16}$

$d = 4$

$Three\ numbers = a - d, a, a + d$

$= 6 - 4, 6, 6 + 4$

$= 2, 6, 10.$

26)The sum and product of three consecutive terms of an AP are respectively 21 and 280. Find the numbers.

27)Three numbers are in AP. Their sum is 15 and the product of the extremes is 21. Find the numbers.

28)The seventh term of an arithmetic progression is four times its second term and twelfth term is 2 more than three times of its fourth term. Find the progression.

Solution :

$a_7 = 4a_2$

$a + 6d = 4(a + d)$

$a + 6d = 4a + 4d$

$6d - 4d = 4a - a$

$2d = 3a ....(1)$

$a_{12} = 2 + 3a_4$

$a + 11d = 2 + 3(a + 3d)$

$a + 11d = 2 + 3a + 9d$

$11d - 9d = 3a - a + 2$

$2d = 2a + 2$

$3a = 2a + 2$

$3a - 2a = 2$

$a = 2$

$Substitute\ in\ (1)$

$2d = 3\ (2)$

$d = 3$

$AP : 2, 5, 8, 11, ...$

29) There are 5 terms in an Arithmetic Progression. The sum of these terms is 55 and the fourth term is five more than the sum of the first two terms. Find the terms of the Arithmetic Progression.

Solution :

$a_1 + a_2 + a_3 + a_4 + a_5 = 55$

$a + a + d + a + 2d + a + 3d + a + 4d = 55$

$5a + 10d = 55$

$5(a + 2d) = 55$

$a + 2d = \frac{55}{5}$

$a + 2d = 11 \ldots\ldots \ldots\ldots \ldots (1)$

$a_4 = 5 + a_1 + a_2$

$a + 3d = 5 + a + a + d$

$a + 3d = 5 + 2a + d$

$2a - a + d - 3d = -5$

$a - 2d = -5 \ldots\ldots\ldots \ldots (2)$

$(1) + (2)\ gives$

$a + 2d = 11$

$\underline{a - 2d = -5}$

$2a = 6$

$a = \frac{6}{2}$

$a = 3$

$Substitute\ in\ (1)$

$a + 2d = 11$

$3 + 2d = 11$

$2d = 11 - 3$

$2d = 8$

$d = \frac{8}{2}$

$d = 4$

$AP\ is\ 3, 7, 11, \ldots \ldots$

30) In an Arithmetic Progression, sixth term is one more than twice the third term. The sum of the fourth and fifth terms is 5 times the second term. Find the 10th term of the Arithmetic Progression.

Solution :

$a_6 = 1 + 2a_3$

$a + 5d = 1 + 2(a + 2d)$

$a + 5d = 1 + 2a + 4d$

$2a - a + 4d - 5d = -1$

$a - d = -1 \ldots\ldots \ldots \ldots (1)$

$a_4 + a_5 = 5a_2$

$a + 3d + a + 4d = 5(a + d)$

$2a + 7d = 5a + 5d$

$5a - 2a + 5d - 7d = 0$

$3a - 2d = 0 \ldots\ldots\ldots \ldots (2)$

$(2) - (1) \times 2\ gives$

$3a - 2d = 0$

$\underline{2a - 2d = -2}$

$a = 2$

$Substitute\ in\ (1)$

$a - d = -1$

$2 - d = -1$

$-d = -1 - 2$

$-d = -3$

$d = 3$

$a_{10} = a + 9d$

$= 2 + 9(3)$

$= 2 + 27$

$= 29$

31)If sum of the four consecutive terms of an Arithmetic Progression is 32 and the ratio of the product of the first and the last term to the product of the middle two terms is 7 : 15, find the terms.

Solution :

$Let\ the\ four\ cosecutive\ terms\ be$

$a - 3d,\ \ a - d, a + d\ and\ a + 3d.$

$Sum = 32$

$a - 3d + a - d + a + d + a + 3d = 32$

$4a = 32$

$a = \frac{32}{4}$

$a = 4$

$Ratio = \frac{7}{15}$

$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$

$\frac{8^2-9d^2}{8^2-d^2} = \frac{7}{15}$

$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$

$15(64-9d^2) = 7(64-d^2)$

$960 - 135d^2 = 448 - 7d^2$

$135d^2 - 7d^2 = 960 - 448$

$128d^2 = 512$

$d^2 = \frac{512}{128}$

$d^2 = 4$

$d = \sqrt{4}$

$d = 2$

$\therefore Terms\ are = a-3d, a-d, a+d, a+3d$

$= 8-3(2), 8-2, 8+2, 8+3(2)$

$= 8-6, 6, 10, 8+6$

$= 2,\ 6,\ 10,\ 14$

32)An arithmetic progression consists 50 terms. The sum of the first 10 terms of it is 210 and the sum of the last 15 terms is 2565, then find the arithmetic progression.

Solution :

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}[2a+(10-1)d]$

$210 = 5[2a+9d]$

$2a+9d = \frac{210}{5}$

$2a+9d = 42 \ldots\ldots\ldots\ldots(1)$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{15} = \frac{15}{2}[2(a_{36})+(15-1)d]$

$2565 = \frac{15}{2}[2(a+35d)+14d]$

$2565 = \frac{15}{2}[2a+70d+14d]$

$2565 = \frac{15}{2}[2a+84d]$

$2a+84d = \frac{2565 \times 2}{15}$

$2a+84d = 171 \times 2$

$2a+84d = 342 \ldots\ldots\ldots\ldots(2)$

$(2)-(1)\ gives$

$2a+84d = 342$

$2a+9d = 42$

$75d = 300$

$d = \frac{300}{75}$

$d = 4$

$Substitute\ in\ (1)$

$2a+9d = 42$

$2a+9(4) = 42$

$2a+36 = 42$

$2a = 42-36$

$2a = 6$

$a = \frac{6}{2}$

$a = 3$

$\therefore AP\ is\ 3, 7, 11, \ \ldots\ldots$

Lesson-2. Triangles.

Similar triangles :

Two triangles are said to be similar, if

i) Their corresponding angles are equal.

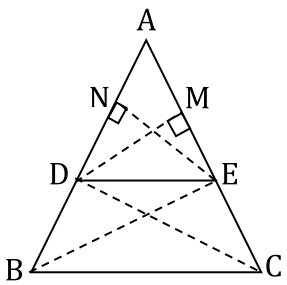
ii) Their corresponding sides are proportional.

All circles, all squares and all equilateral triangles are always similar.

Theorem 1 :

(**Thales Theorem OR Basic Proportionality Theorem)** :

"A line drawn parallel to one side of a triangle divides the other two sides in the same ratio."



Data : In Δ ABC, DE || BC.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction : Draw DM ⊥ AC and EN ⊥ AB.

Proof :

$$\frac{Area\ of\ \Delta\ ADE}{Area\ of\ \Delta\ BDE} = \frac{\frac{1}{2}\ X\ AD\ X\ EN}{\frac{1}{2}\ X\ DB\ X\ EN}$$

$$= \frac{AD}{DB} \ldots\ldots\ldots\ldots\ldots(1)$$

$$\frac{Area\ of\ \Delta\ ADE}{Area\ of\ \Delta\ DEC} = \frac{\frac{1}{2}\ X\ AE\ X\ DM}{\frac{1}{2}\ X\ EC\ X\ DM}$$

$$= \frac{AE}{EC} \ldots\ldots\ldots\ldots\ldots(2)$$

Δ BDE and Δ DEC stand on the same base DE and between the same parallel lines DE and BC.

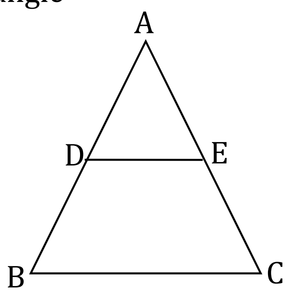
$\therefore Area\ of\ \Delta\ BDE = Area\ of\ \Delta\ DEC$

So from equations (1) and (2), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Corollary of BPT :

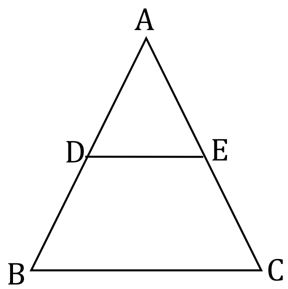
"If a line is drawn parallel to one side of a triangle, then the sides of the new triangle formed are proportional to the sides of the given triangle"



In Δ ABC, if DE || BC, then $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

Converse of BPT :

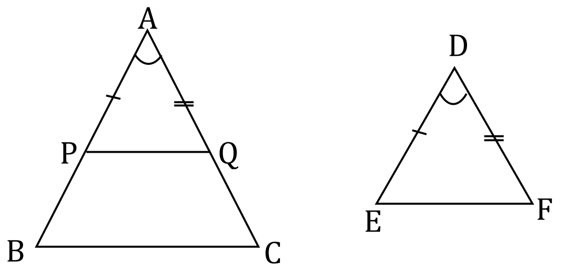
"If a line divides any two sides of a triangle in the same ratio, then that line is parallel to the third side".



In Δ ABC, if $\frac{AD}{DB} = \frac{AE}{EC}$ then DE || BC, then

Theorem 2 (AA Criterion) :

"If the corresponding angles of two triangles are equal, then their corresponding sides are in the same ratio".



Data : In Δ ABC and Δ DEF,

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

To Prove : $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Construction : Mark the points P and Q on AB and AC such that AP = DE and AQ = DF.

Proof :

In Δ APQ and Δ DEF,

$\angle A = \angle D$ [By data]

AP = DE [By Construction]

AQ = DF [By construction]

$\therefore \Delta APQ \cong \Delta DEF$ [SAS congruence]

$\therefore \angle P = \angle E$ *and* $PQ = EF$ $[C.P.C.T]$

Now $\angle B = \angle E$.

$\therefore \angle P = \angle B$ and PQ ∥ BC.

$\therefore \frac{AB}{AP} = \frac{AC}{AQ} = \frac{BC}{PQ}$ [*Corollary of B.P.T*]

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ [*By Substitution*]

Hence proved.

SSS Criterion :

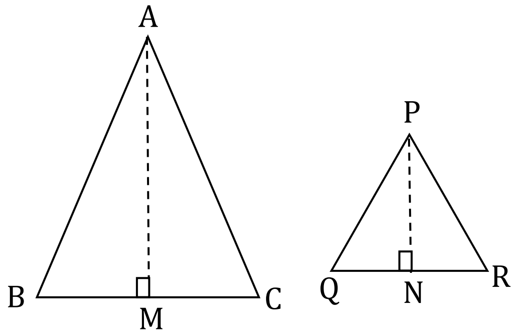
"If the corresponding sides of two triangles are proportional, then their corresponding angles are equal".

SAS Criterion :

"If one pair of corresponding angles of two triangles are equal and their included sides are proportional, then the two triangles are similar".

Theorem 3 (Areas of Similar Triangles]

"The areas of two similar triangles are proportional to the squares of their corresponding sides".



Data : $\Delta ABC \sim \Delta PQR$.

$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

To Prove : $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta PQR} = \frac{BC^2}{QR^2}$.

Construction : Draw AL ⊥ BC and PM ⊥ QR.

Proof :

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta PQR} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$$

$$= \frac{BC}{QR} \times \frac{AM}{PN} \ \ldots\ldots\ldots\ (1)$$

In Δ ABM and Δ PQN,

$\angle B = \angle Q$ [By data]

$\angle M = \angle N$ [Right angles]

$\therefore \Delta ABM \sim \Delta PQN$ [AA Criterion]

$\therefore \frac{AB}{PQ} = \frac{BM}{QN} = \frac{AM}{PN}$

But $\frac{AB}{PQ} = \frac{BC}{QR}$

$\therefore \frac{AM}{PN} = \frac{BC}{QR}$ .... (2)

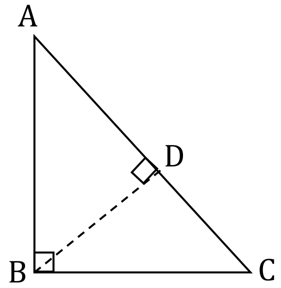
Substitute (2) in (1)

$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta PQR} = \frac{BC^2}{QR^2}$

Hence proved.

Theorem 4 (Pythagorus Theorem) :

"In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides".



Data : ABC is a right angled triangle at B.

To Prove : $AC^2 = AB^2 + BC^2$.

Construction : Draw $BD \perp AC$.

Proof :

In Δ ABC and Δ ADB,

$\angle A = \angle A$ [Common angles]

$\angle B = \angle D$ [Right angles]

$\therefore \Delta ABC \sim \Delta ADB$ [AA Criterion]

$\therefore \frac{AB}{AD} = \frac{BC}{DB} = \frac{AC}{AB}$

$AB^2 = AC.AD$. . . . . (1)

In Δ ABC and Δ BDC,

$\angle C = \angle C$ [Common angles]

$\angle B = \angle D$ [Right angles]

$\therefore \Delta ABC \sim \Delta BDC$ [AA Criterion]

$\therefore \frac{AB}{BD} = \frac{BC}{DC} = \frac{AC}{BC}$

$BC^2 = AC.DC$. . . . . (2)

Adding equations (1) and (2),

$AB^2 + BC^2 = AC.AD + AC.DC$

$= AC(AD + DC)$

$= AC.AC$

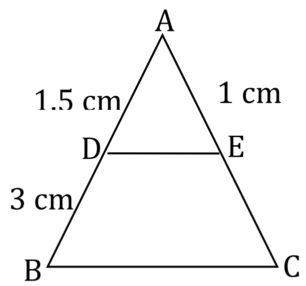
$= AC^2$.

Converse of Pythagorus Theorem :

"In a triangle, if the square of one side is equal to the sum of the squares of other two sides, then those two sides contain a right angle."

Problems :

1) In Δ ABC, DE || BC. If AD = 1.5 cm, BD = 3 cm, AE = 1 cm, find EC.



Solution :

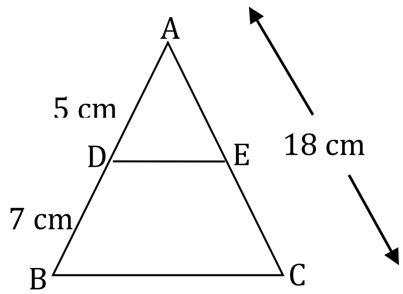
By Thales theorem, we have

$\frac{EC}{AE} = \frac{DB}{AD}$

$\frac{EC}{1} = \frac{3}{1.5}$

EC = 2 cm

2) In Δ ABC, DE || BC, AD = 5 cm, BD = 7 cm and AC = 18 cm. Find AE and EC.



Solution :

By corollary of B.P.T, we have

$\frac{AE}{AC} = \frac{AD}{AB}$

$\frac{AE}{18} = \frac{5}{12}$

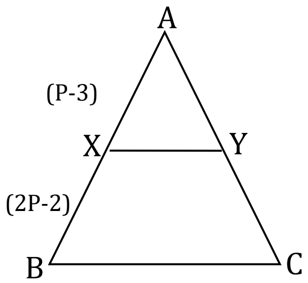
$AE = \frac{5}{12} \times 18$

$= 7.5$ cm.

3) In Δ ABC, DE || AB, AD = 7 cm, CD = 5 cm and BC = 18 cm, then find CE.

4) In Δ PQR, ST || QR, PS = 3 cm, PT = 5 cm and QS = 6 cm. Find TR.

5) In Δ ABC, XY || BC, AX = (p – 3), BX = 2p – 2 and $\frac{AY}{CY} = \frac{1}{4}$, find p.



<u>Solution:</u>

By B.P.T, we have

$$\frac{AX}{BX} = \frac{AY}{CY}$$

$$\frac{P-3}{2P-2} = \frac{1}{4}$$

4(P – 3) = 1(2P – 2)

4P – 12 = 2P – 2.

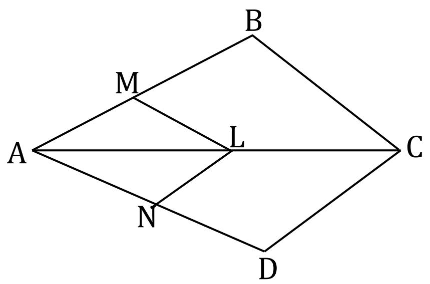
4P – 2P = -2 + 12

2P = 10

$$P = \frac{10}{2}$$

P = 5 cm.

6) In the figure, if LM || CB and LN || CD, $\frac{AM}{AB} = \frac{AN}{AD}$.



<u>Solution</u> :

In Δ ABC, LM || CB.

∴ By BPT, we have

$$\frac{AM}{AB} = \frac{AL}{AC} \text{. . . . . (1)}$$

In Δ ADC, LN || CD.

∴ By BPT, we have

$$\frac{AN}{AD} = \frac{AL}{AC} \text{. . . . . (2)}$$

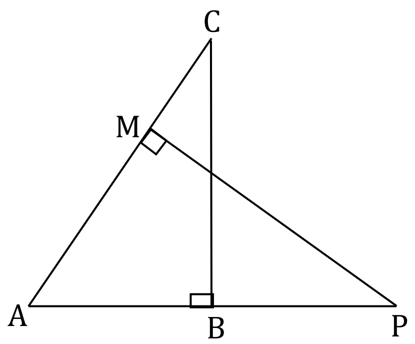
From (1) and (2), we have

$$\frac{AM}{AB} = \frac{AN}{AD}$$

7) In the figure, ABC and AMP are two right triangles at B and M respectively. Prove that

(*i*) Δ *ABC* ~Δ*AMP*

(*ii*) $\frac{CA}{PA} = \frac{BC}{MP}$



<u>Solution :</u>

In Δ*ABC and* Δ*AMP*,

$\angle A = \angle A$ [Common angles]

$\angle B = \angle B$ [Right angles]

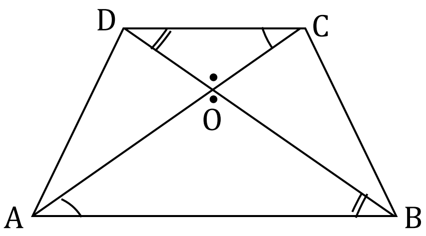
∴ $\Delta ABC \sim \Delta AMP$ [AA criterion]

$$\therefore \frac{AB}{AM} = \frac{BC}{MP} = \frac{AC}{AP} \quad [corresponding\ sides]$$

$$\frac{CA}{PA} = \frac{BC}{MP}$$

Hence proved.

8) In a trapezium ABCD, AB || CD, diagonals C and BD intersect at O. Prove that AO. OD = BO. OC.



Solution :

In Δ AOB and Δ COD,

∠A = ∠C [Alternate angles]

∠B = ∠D [Alternate angles]

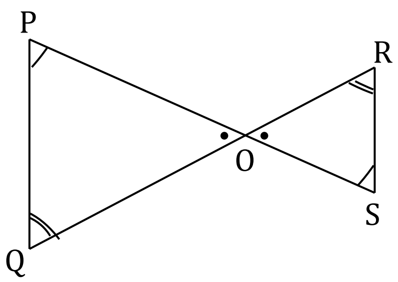
$\therefore \Delta\, AOB \sim \Delta\, COD$ [AA Criterion]

$\therefore \frac{AO}{CO} = \frac{OB}{OD} = \frac{AB}{CD}$

AO. OD = OB. CO

Hence proved.

9) In the figure if PQ ∥ RS, prove that $\Delta POQ \sim SOR$.



Solution :

In Δ POQ and Δ SOR,

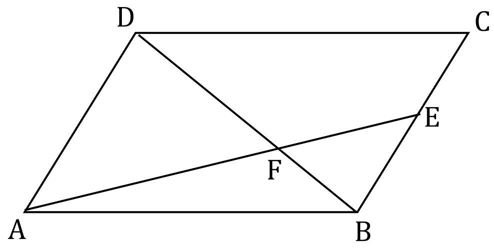
∠P = ∠S [Alternate angles]

∠Q = ∠R [Alternate angles]

$\therefore \Delta\, POQ \sim \Delta\, SOR$ [AA Criterion]

Hence proved.

10)The diagonal BD of parallelogram ABCD intersects AE at F as shown in the figure. If E is any point on BC, then prove that $DF \times EF = FB \times FA$.



Solution :

In Δ AFD and Δ EFB,

∠A = ∠E [Alternate angles]

∠F = ∠F [V. O. A]

$\therefore \Delta\, AFD \sim \Delta\, EFB$ [AA Criterion]

$\therefore \frac{AF}{EF} = \frac{FD}{FB} = \frac{AD}{EB}$

$\therefore DF \times EF = FB \times FA$

Hence Proved.

11)Let $\Delta\, ABC \sim \Delta\, DEF$ and their areas be $64 cm^2$ and $121 cm^2$ respectively. If EF = 15.4 cm, find BC.

Solution :

$$\frac{Area\ of\ \Delta\, ABC}{Area\ of\ \Delta\, DEF} = \frac{BC^2}{EF^2}$$

$$\frac{64}{121} = \frac{BC^2}{EF^2}$$

$$\frac{BC}{EF} = \sqrt{\frac{64}{121}}$$

$$\frac{BC}{EF} = \frac{8}{11}$$

$$\frac{BC}{15.4} = \frac{8}{11}$$

$$BC = \frac{8}{11} \times 15.4$$

$$= 8 \times 1.4$$

$$= 11.2 \text{ cm}$$

12)Let $\Delta\, ABC \sim \Delta\, DEF$ and their areas be $64 cm^2$ and $100\ cm^2$ respectively. If EF = 12 cm, find BC.

13)Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio,

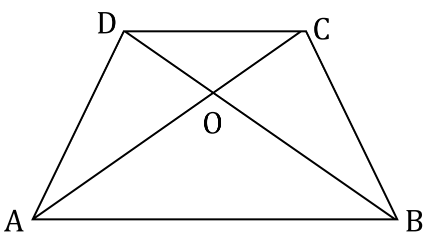
Solution :

$$\frac{Area\ of\ \Delta\, ABC}{Area\ of\ \Delta\, DEF} = \frac{BC^2}{EF^2}$$

$$= \left(\frac{4}{9}\right)^2$$

$$= \frac{16}{81}$$

14)Diagonals of a trapezium ABCD with AB|| DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.



Solution :

$\Delta\, AOB \sim \Delta\, COD$

$$\frac{Area\ of\ \Delta\, AOB}{Area\ of\ \Delta\, COD} = \frac{AB^2}{CD^2}$$

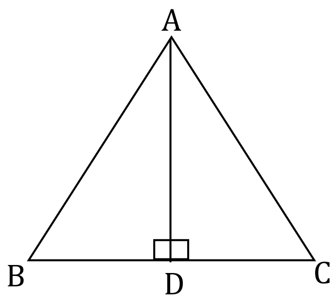
$$\frac{Area\ of\ \Delta\, AOB}{Area\ of\ \Delta\, COD} = \frac{(2CD)^2}{CD^2}$$

$$= \frac{4CD^2}{CD^2}$$

$$= \frac{4}{1}$$

$$= 4 : 1.$$

15)In Δ ABC, AD ⊥ BC and $AD^2 = BD \times CD$. Prove that $AB^2 + AC^2 = (BD + CD)^2$.



Solution :

By Pythagorus theorem,

$AB^2 = AD^2 + BD^2$ .... (1)

$AC^2 = AD^2 + CD^2$ .... (2)

Adding (1) and (2),

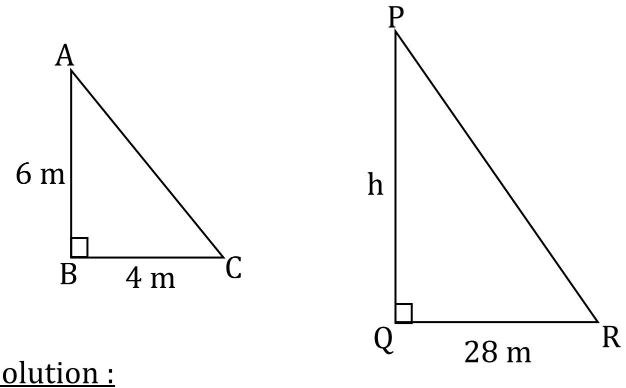
$AB^2 + AC^2 = AD^2 + BD^2 + AD^2 + CD^2$

$= BD^2 + 2AD^2 + CD^2$

$= BD^2 + 2 \times BD \times CD + CD^2$

$= (BD + CD)^2$.

16)A vertical pole of height 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.



Solution :

$\Delta\, ABC \sim \Delta\, PQR$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\frac{6}{PQ} = \frac{4}{28}$$

$$4 \times PQ = 6 \times 28$$

$$PQ = \frac{6 \times 28}{4}$$

$$= 42\ m$$

Lesson-3. Pair of Linear Equations.

General form of a pair of linear equations :

The general form of a pair of linear equations in two variables x and y is,

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Solution of a pair of linear equations :

The value of x and y that satisfy the pair of linear equations is called solution of the equations.

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then

i) The equations have **a unique solution.**

ii) The graph of the equations is **intersecting lines.**

iii) The system of equations is **consistent.**

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then

i) The equations have **infinitely many solutions.**

ii) The graph of the equations is **coincident lines.**

iii) The system of equations is **consistent.**

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then

i) The equations have **no solution.**

ii) The graph of the equations is **parallel lines.**

iii) The system of equations is **inconsistent.**

1) Find out whether the equations are consistent or inconsistent.

$\mathbf{3x + 2y = 5}$

$\mathbf{2x - 3y = 7}$

Solution :

$3x + 2y = 5$

$2x - 3y = 7$

Here, $\frac{a_1}{a_2} = \frac{3}{2}$

$\frac{b_1}{b_2} = \frac{2}{-3}$

$\frac{c_1}{c_2} = \frac{5}{7}$

Now, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ Unique solution.

Consistent.

Intersecting lines.

2) Find out whether the equations are consistent or inconsistent.

$\mathbf{2x - 3y = 8}$

$\mathbf{4x - 6y = 9}$

Solution :

$2x - 3y = 8$

$4x - 6y = 9$

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{8}{9}$

Now, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ No solution.

Inconsistent.

Parallel lines.

3) Find out whether the equations are consistent or inconsistent.

$\mathbf{5x - 3y = 11}$

$\mathbf{-10x + 6y = -22}$

Solution :

$5x - 3y = 11$

$-10x + 6y = -22$

Here, $\frac{a_1}{a_2} = \frac{5}{-10} = \frac{1}{-2}$

$\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$

$\frac{c_1}{c_2} = \frac{11}{-22} = \frac{1}{-2}$

Now, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ Infinitely many solutions.

Consistent.

Coincident lines.

4) Find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident.

$(i)\ 5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

$(ii)\ 9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

$(iii) 6x - 3y + 10 = 0$

$2x - y + 9 = 0$

5) Find out whether the following pairs of equations are consistent or inconsistent?

$(i)\ x - 2y = 0$

$3x + 4y - 20 = 0$

$(ii)\ 2x + 3y - 9 = 0$

$4x + 6y - 18 = 0$

$(iii)\ x + 2y - 4 = 0$

$2x + 4y - 12 = 0$

$(iv)\ x + y = 5$

$2x + 2y = 10$

$(v)\ x - y = 8$

$3x - 3y = 16$

$(vi)\ 2x + y - 6 = 0$

$4x - 2y - 4 = 0$

$(vii)\ 2x - 2y - 2 = 0$

$4x - 4y - 5 = 0$

6) *Solve* : $3x + 2y = 11$

$5x - 2y = 13$.

Solution :

$3x + 2y = 11 \ldots\ldots\ldots (1)$

$5x - 2y = 13 \ldots\ldots\ldots (2)$

$(1) + (2)$ *gives*

$5x - 2y = 13$

$3x + 2y = 11$

$8x = 24$

$x = \frac{24}{8}$

$= 3$

Put $x = 3$ *in equation* (1),

$3x + 2y = 11$

$3(3) + 2y = 11$

$9 + 2y = 11$

$2y = 11 - 9$

$2y = 2$

$y = \frac{2}{2}$

$= 1.$

7) *Solve* : $4x - 2y = -17$

$4x + 2y = 23$.

Solution :

$4x - 2y = -17 \ldots\ldots\ldots (1)$

$4x + 2y = 23 \ldots\ldots\ldots (2)$

$(1) + (2)$ *gives*

$$4x - 2y = -17$$
$$4x + 2y = 23$$
$$8x = 6$$
$$x = \frac{6}{8}$$
$$= \frac{3}{4}$$

$Put\ x = \frac{3}{4}\ in\ equation\ (1)$

$$4x - 2y = -17$$
$$4\left(\frac{3}{4}\right) - 2y = -17$$
$$3 - 2y = -17$$
$$3 + 17 = 2y$$
$$2y = 20$$
$$y = \frac{20}{2}$$
$$y = 10$$

8) *Solve* : $2x + y = 6$

$x - y = 3.$

Solution :

$2x + y = 6 \ldots\ldots\ldots\ldots(1)$

$x - y = 3 \ldots\ldots\ldots\ldots\ldots(2)$

$(1) + (2)\ gives$

$$2x + y = 6$$
$$x - y = 3$$
$$3x = 9$$
$$x = \frac{9}{3}$$
$$x = 3$$

$Put\ x = 3\ in\ equation\ (1)$

$$2x + y = 6$$
$$2(3) + y = 6$$
$$6 + y = 6$$
$$y = 6 - 6$$
$$y = 0$$

9) *Solve* : $3x + 2y - 7 = 0$

$4x + y - 6 = 0$

Solution :

$3x + 2y = 7 \ldots\ldots\ldots\ldots\ldots\ldots(1)$

$4x + y = 6 \ldots\ldots\ldots\ldots\ (2)$

$(2) \times 2 - (1), gives$

$$8x + 2y = 12$$
$$3x + 2y = 7$$
$$5x = 5$$
$$x = \frac{5}{5}$$
$$= 1.$$

$Put\ x = 1\ in\ equation\ (1)$

$$3x + 2y = 7$$
$$3(1) + 2y = 7$$
$$2y = 7 - 3$$
$$2y = 4$$
$$y = \frac{4}{2}$$
$$= 2.$$

10) *Solve* : $3x + y = 15$

$2x - y = 5$

11) *Solve* : $x + y = 8$

$2x - y = 7$

12) For what values of k will the following pair of linear equations have infinitely many solutions?

$kx + 3y - (k - 3) = 0$

$12x + ky - k = 0$

Solution :

$kx + 3y - (k - 3) = 0$

$12x + ky - k = 0$

Equations have infinitely many solutions.

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{12} = \frac{3}{k} = \frac{(k-3)}{k}$$

Consider, $\frac{k}{12} = \frac{3}{k}$

$k^2 = 36$

$k = \pm\sqrt{36}$

$k = \pm 6 \ldots\ldots\ldots\ldots (1)$

Consider, $\frac{3}{k} = \frac{(k-3)}{k}$

$3 = (k-3)$

$k = 3+3$

$k = 6 \ldots\ldots\ldots\ldots (2)$

From (1) *and* (2) $k = 6$.

13) For which values of p does the pair of equations given below have unique solution?

$\mathbf{4x + py + 8 = 0}$

$\mathbf{2x + 2y + 2 = 0}$

Solution :

$4x + py + 8 = 0$

$2x + 2y + 2 = 0$

Equations have unique solution.

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{4}{2} \neq \frac{p}{2}$

$4 \neq p$

OR $p \neq 4$

14)For what value of 'K' will the following pair of linear equations have no solution.

$\mathbf{3x + y = 1}$

$\mathbf{(2k - 1)x + (k - 1)y = 2k + 1}$.

Solution :

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

Given equations have no solution.

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$

Consider, $\frac{3}{2k-1} = \frac{1}{k-1}$

$2k - 1 = (k - 1)3$

$2k - 1 = 3k - 3$

$3k - 3 = 2k - 1$

$3k - 2k = -1 + 3$

$k = 2 \ldots\ldots\ldots\ldots (1)$

Consider, $\frac{1}{k-1} \neq \frac{1}{2k+1}$

$2k + 1 \neq k - 1$

$2k - k \neq -1 - 1$

$k \neq -2 \ldots\ldots\ldots\ldots (2)$

$\therefore$ *from* (1) *and* (2) $k = 2$.

15)Solve graphically : $2x + y = 5$

$x + y = 4$

Solution :

x	0	1
y	5	3

$2x + y = 5$

$2(0) + y = 5$

$y = 5$

$2x + y = 5$

$2(1) + y = 5$

$2 + y = 5$

$y = 5 - 2$

$y = 3$

x	0	1
y	4	3

$x + y = 4$

$0 + y = 4$

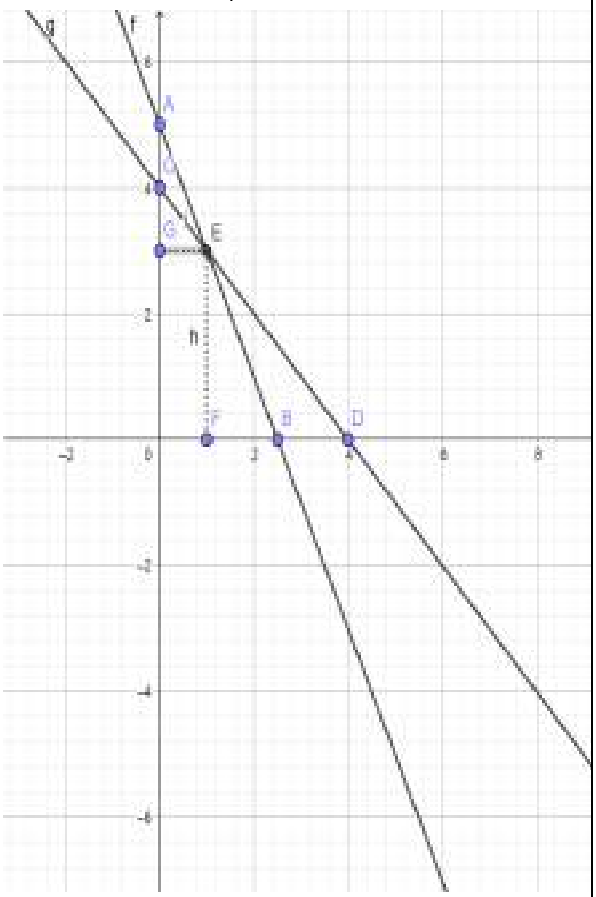
$y = 4$

$x + y = 4$

$(1) + y = 4$

$y = 4 - 1$

$y = 3$



16)Solve graphically : $3x + y = 8$

$2x - y = 2$

Solution :

x	0	1
y	8	5

$3x + y = 8$

$3(0) + y = 8$

$0 + y = 8$

$y = 8$

$3x + y = 8$

$3(1) + y = 8$

$3 + y = 8$

$y = 8 - 3$

$y = 5$

x	0	1
y	-2	0

$2x - y = 2$

$2(0) - y = 2$

$0 - y = 2$

$y = -2$

$2x - y = 2$

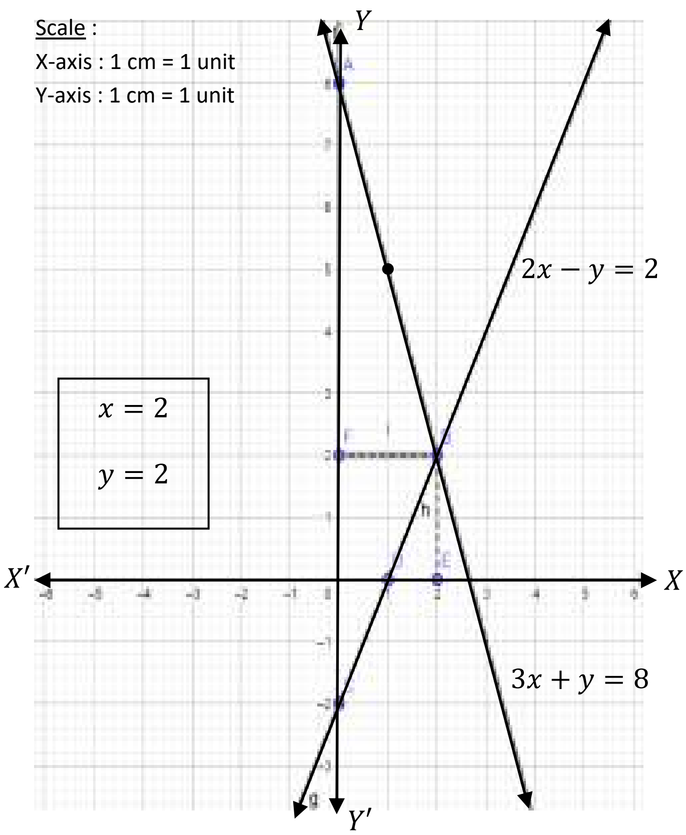
$2(1) - y = 2$

$2 - y = 2$

$-y = 2 - 2$

$-y = 0$

$y = 0$



17) Solve graphically : $x - y + 3 = 0$
$2x - y + 9 = 0$

Solution :

x	0	1
y	3	4

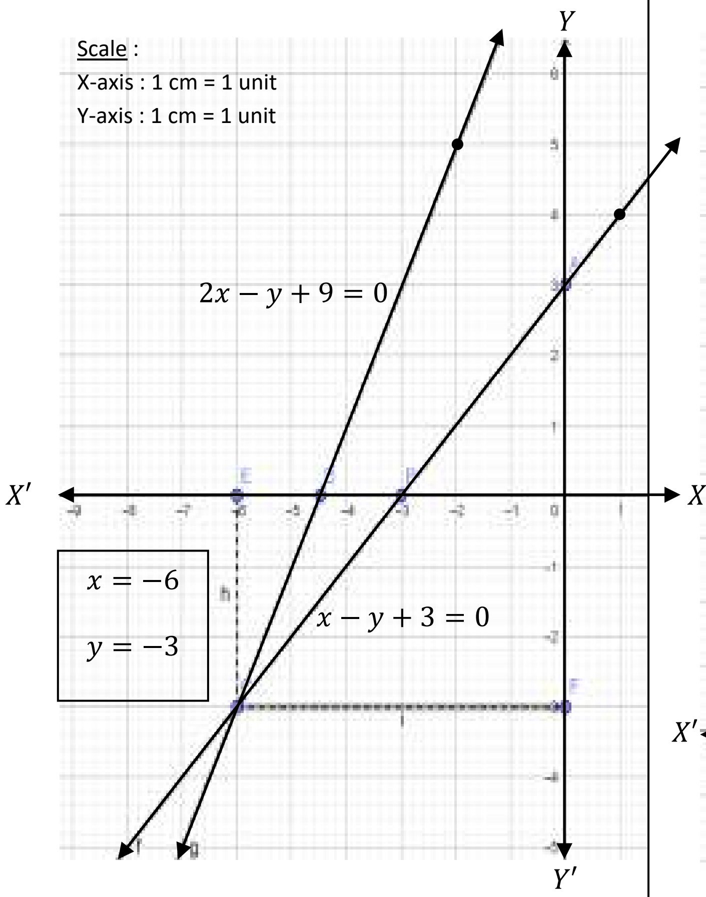
$x - y + 3 = 0$
$0 - y + 3 = 0$
$-y = -3$
$y = 3$

$x - y + 3 = 0$
$(1) - y + 3 = 0$
$1 - y + 3 = 0$
$4 - y = 0$
$y = 4$

x	0	1
y	9	11

$2x - y + 9 = 0$
$2(0) - y + 9 = 0$
$0 - y + 9 = 0$
$-y + 9 = 0$
$y = 9$

$2x - y + 9 = 0$
$2(1) - y + 9 = 0$
$2 - y + 9 = 0$
$11 - y = 0$
$y = 11$



18) Solve graphically : $2x + y = 10$
$x + y = 7$

Solution :

x	0	1
y	10	8

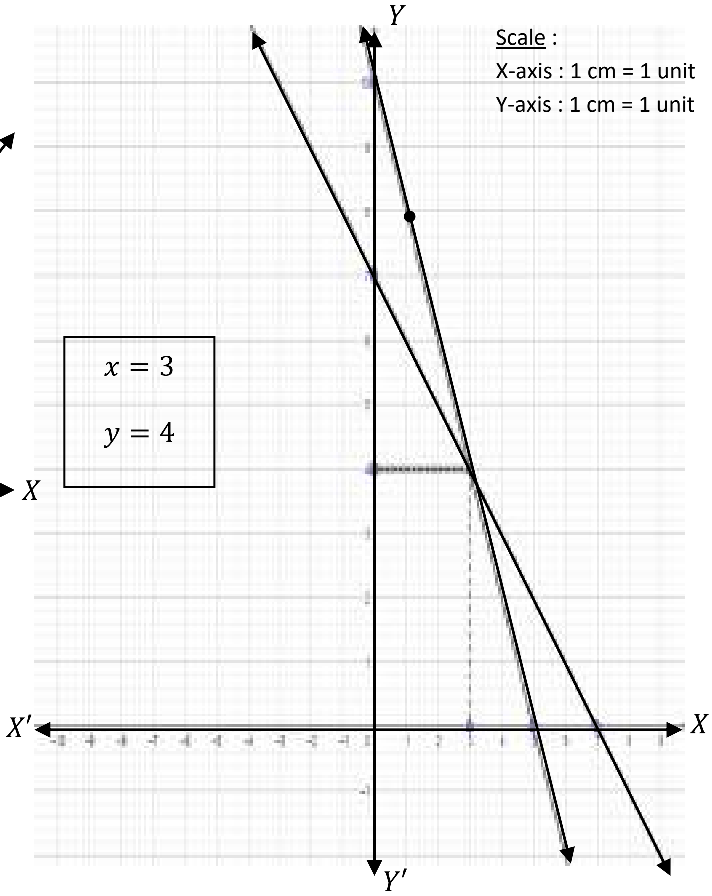
$2x + y = 10$
$2(0) + y = 10$
$0 + y = 10$
$y = 10$

$2x + y = 10$
$2(1) + y = 10$
$2 + y = 10$
$y = 10 - 2$
$y = 8$

x	0	1
y	7	6

$x + y = 7$
$0 + y = 7$
$y = 7$

$x + y = 7$
$(1) + y = 7$
$1 + y = 7$
$y = 7 - 1$
$y = 6$



19)Solve graphically : $2x + y = 6$

$4x + y = 8$

Solution :

x	0	1
y	6	4

x	0	1
y	8	4

$2x + y = 6$

$2(0) + y = 6$

$0 + y = 6$

$y = 6$

$4x + y = 8$

$4(0) + y = 8$

$0 + y = 8$

$y = 8$

$2x + y = 6$

$2(1) + y = 6$

$2 + y = 6$

$y = 6 - 2$

$y = 4$

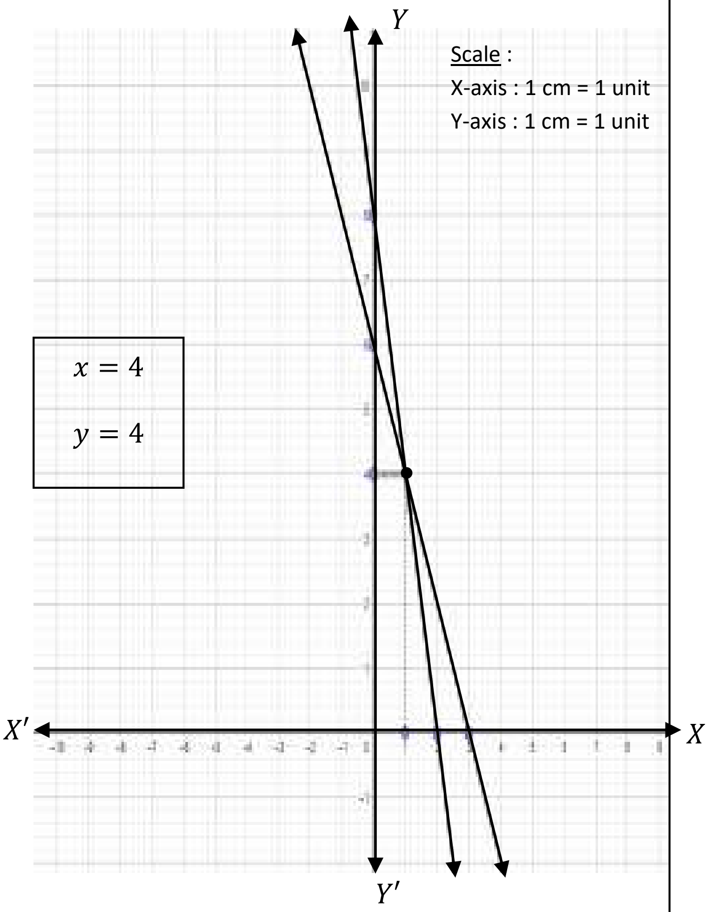
$4x + y = 8$

$4(1) + y = 8$

$4 + y = 8$

$y = 8 - 4$

$y = 4$



$x = 4$

$y = 4$

20)Solve graphically : $2x + y = 6$

$2x - y = 2$

Solution :

x	0	1
y	6	4

x	0	2
y	- 2	0

$2x + y = 6$

$2(0) + y = 6$

$0 + y = 6$

$y = 6$

$2x - y = 2$

$2(0) - y = 2$

$0 - y = 2$

$y = -2$

$2x + y = 6$

$2(1) + y = 6$

$2 + y = 6$

$y = 6 - 2$

$y = 4$

$2x - y = 2$

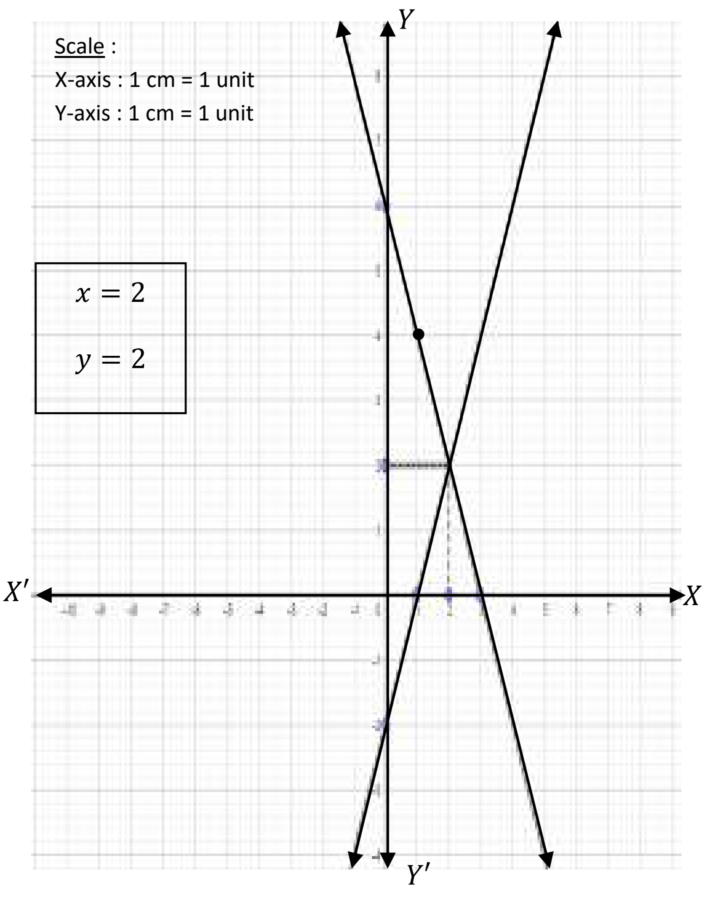
$2(1) - y = 2$

$2 - y = 2$

$-y = 2 - 2$

$-y = 0$

$y = 0$



$x = 2$

$y = 2$

21)Solve the following pair of linear equations by graphical method.

$x + y = 7$

$3x - y = 1$

22)Solve the following pair of linear equations by graphical method.

$2x - y - 4 = 0$

$x + y + 1 = 0$

23)Solve the following pair of linear equations by graphical method.

$x + 2y = 5$

$2x - 3y = 6$

24)Solve the following pair of linear equations by graphical method.

$x + y = 5$

$2x - y = 4$

25)The difference between two numbers is 26 and one number is three times the other. Find them.

<u>Solution</u> :

Let the two numbers be x and y.

$\therefore\ x - y = 26 \ldots\ldots\ldots.(1)$

$and\ x = 3y$

$or\ x - 3y = 0 \ldots\ldots\ldots..(2)$

$(1) - (2)\ gives,$

$x - y = 26$

$\underline{x - 3y = 0}$

$2y = 26$

$$y = \frac{26}{2}$$

$y = 13.$

$Substitute\ in\ (1)$

$x - y = 26$

$x - 13 = 26$

$x = 26 + 13$

$= 39$

$\therefore$ *The required numbers are* 39 *and* 13.

26)Raju can row downstream 20 km in 2 hour and upstream 4 km in 2 hours. Find his speed of rowing in still water and the speed of the current.

<u>*Solution :*</u>

$Let\ speed\ of\ the\ current = x\ km/hr.$

$Let\ speed\ of\ rowing\ in\ still\ water$

$= y\ km/hr$

$Speed\ downstream = x + y.$

$Speed\ upstream = x - y.$

$Distance\ travelled\ downstream$

$= 20\ km.$

$Distance\ travelled\ upstream = 4\ km.$

$$Speed\ downstream = \frac{distance}{time}$$

$$x + y = \frac{20}{2}$$

$x + y = 10 \ldots\ldots\ldots(1)$

$$Speed\ upstream = \frac{distance}{time}$$

$$x - y = \frac{4}{2}$$

$x - y = 2 \ldots\ldots\ldots..\ (2)$

$(1) + (2)\ gives,$

$x + y = 10$

$\underline{x - y = 2}$

$2x = 12$

$$x = \frac{12}{2}$$

$x = 6.$

$Substitute\ in\ (1)$

$x + y = 10$

$6 + y = 10$

$y = 10 - 6$

$y = 4$

$\therefore Speed\ of\ the\ current = 6\ km/hr.$

$Speed\ of\ rowing\ in\ still\ water$

$= 4\ km/hr.$

27)Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nun will be twice as old as Sonu. How old are Nun and Sonu?

Solution :

$Let\ the\ present\ Age\ of\ Nuri = x.$

$Let\ the\ present\ Age\ of\ Sonu = y.$

$5\ years\ ago, Age\ of\ Nuri = x - 5.$

$Age\ of\ Sonu = y - 5.$

$\therefore (x-5) = 3(y-5)$

$x - 5 = 3y - 15.$

$x - 3y = -15 + 5$

$x - 3y = -10 \ldots\ldots\ldots\ldots\ldots\ldots(1)$

$10\ years\ later, Age\ of\ Nuri = x + 10.$

$Age\ of\ Sonu = y + 10.$

$\therefore (x+10) = 2(y+10)$

$x + 10 = 2y + 20.$

$x - 2y = 20 - 10$

$x - 2y = 10 \ldots\ldots\ldots\ldots\ldots\ \ldots\ldots(2)$

$(1) - (2)\ gives,$

$x - 3y = -10$

$\underline{x - 2y = 10}$

$-y = -20$

$y = 20$

$Substitute\ in\ (1)$

$x - 3y = -10$

$x - 3(20) = -10$

$x - 60 = -10$

$x = -10 + 60$

$x = 50.$

$\therefore Age\ of\ Nuri\ is\ 50\ years\ and\ age\ of\ Sonu\ is$ $20\ years.$

28)The sum of the numerator and the denominator of a given fraction is 12. If 3 is added to its denominator, then the fraction becomes ½ . Find the given fraction.

Solution :

$Let\ the\ given\ fraction = \frac{x}{y}.$

$\therefore x + y = 12 \ldots\ldots\ldots\ldots(1)$

$\frac{x}{y+3} = \frac{1}{2}$

$2x = y + 3.$

$2x - y = 3 \ldots\ldots\ldots\ldots(2)$

$(1) + (2)\ gives$

$x + y = 12$

$\underline{2x - y = 3}$

$3x = 15$

$x = \frac{15}{3}$

$x = 5$

$Substitute\ in\ (1)$

$x + y = 12.$

$5 + y = 12$

$y = 12 - 5$

$y = 7$

$\therefore The\ given\ fraction\ is\ \frac{5}{7}$

29)Seven times a two digit number is equal to four times the number obtained by reversing the places of its digits. If the difference between the digits is 3, then find the number.

Solution :

$Let\ the\ digit\ in\ units\ place = x.$

$Let\ the\ digit\ in\ tens\ place = y.$

$The\ original\ number\ =\ 10y\ +\ x.$

$The\ reversed\ number\ =\ 10x\ +\ y.$

$\therefore\ 7(10y\ +\ x)\ =\ 4(10x\ +\ y)$

$70y\ +\ 7x\ =\ 40x\ +\ 4y$

$40x - 7x\ +\ 4y - 70y\ =\ 0$

$33x - 66y\ =\ 0$

$Divide\ by\ 33$

$x - 2y\ =\ 0 ....(1)$

$Difference\ between\ the\ digits\ is\ 3$

$\therefore\ x - y\ =\ 3\ ....(2)$

$(1) - (2)\ gives$

$x\ -\ 2y\ =\ 0$

$\underline{x - y\ =\ 3}$

$-y\ =\ -3$

$y\ =\ 3$

$Substitute\ in\ (2)$

$x - y\ =\ 3$

$x - 3\ =\ 3$

$x\ =\ 3\ +\ 3$

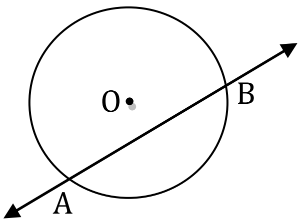
$=\ 6.$

$\therefore\ The\ required\ number\ is\ 63.$

Lesson - 4. Circles

Secant of a circle :

A secant of a circle is a line that intersects the circle at two distinct points.

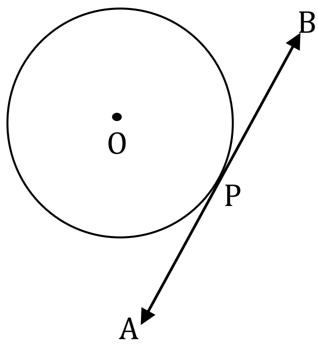


O is the centre of the circle.

AB is a secant of the circle.

Tangent to a circle :

A tangent to a circle is a line that intersects the circle at only one point.

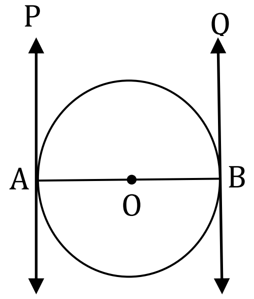


O is the centre of the circle.

AB is a tangent.

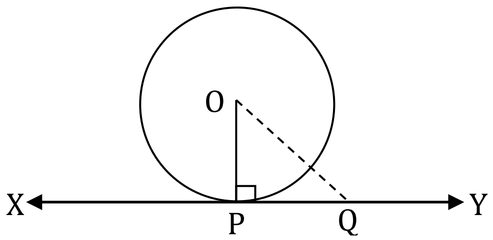
There is only **one tangent** at a point on the circle.

Tangents drawn at the ends of a diameter are parallel.



Theorem 4 :

"The tangent at any point of a circle is perpendicular to the radius drawn at the point of contact".



Data : O is the centre of the circle. XY is the tangent to the circle at the point P. OP is the radius drawn at the point of contact P.

To Prove : $OP \perp XY$.

Construction : Take a point Q on XY. Join OQ.

Proof :

The point Q lies outside the circle.

∴ OQ is longer than OP.

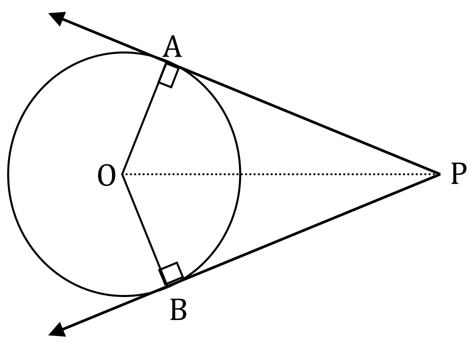
So, OP is the smallest distance of the point O from the line XY.

∴ $OP \perp XY$.

Hence proved.

Theorem 5 :

"The two tangents drawn from an external point to a circle are equal".



Data : O is the centre of the circle. P is an external point. AP and BP are tangents to the circle.

To Prove : AP = BP.

Proof :

In Δ AOP and Δ BOP,

∠OAP = ∠OBP [Right angles]

OA = OB [Radii of the same circle]

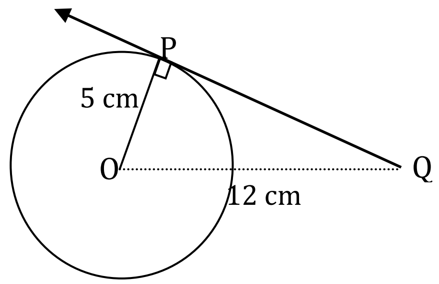
OP = OP [Common side]

∴ Δ AOP ≅ Δ BOP [RHS Theorem]

∴ AP = BP [C.S.C.T]

Hence proved.

1) **A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q , so that OQ = 12 cm. Find PQ.**



Solution :

By Pythagorus theorem,

$OQ^2 = OP^2 + PQ^2$.

$12^2 = 5^2 + PQ^2$

$144 = 25 + PQ^2$

$PQ^2 = 144 - 25$

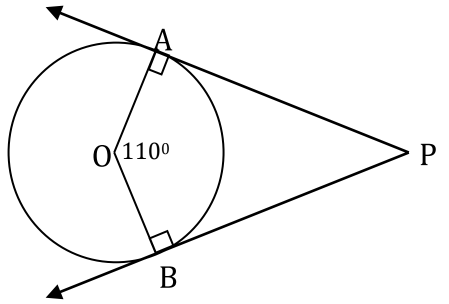
$PQ^2 = 119$

$PQ = \sqrt{119}$ cm

∴ Length of the tangent is $\sqrt{119}$ cm.

2) **From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.**

3) **If TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110^0, then find ∠PTQ.**



Solution :

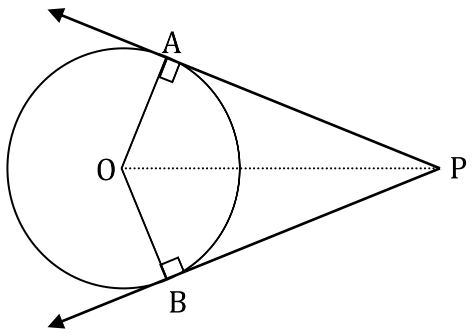
$\angle PTQ + \angle POQ = 180^0$.

$\angle PTQ + 110^0 = 180^0$.

$\angle PTQ = 180^0 - 110^0$.

$= 70^0$.

4) If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80^0, then find $\angle POA$



Solution :

$\angle AOB + \angle APB = 180^0$.

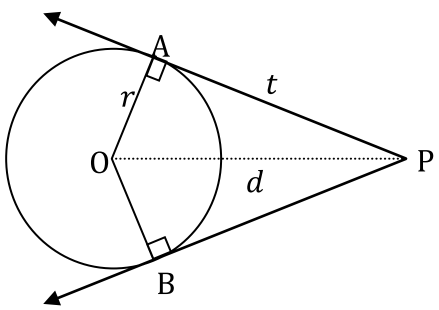
$\angle AOB + 80^0 = 180^0$.

$\angle AOB = 180^0 - 80^0$.

$\angle AOB = 100^0$.

$\therefore \angle POA = 50^0$.

5) The length of a tangent from a point A at a distance 5 cm from the centre of circle is 4 cm. Find the radius of the circle.



Solution :

$t = 4\ cm,$

$d = 5\ cm,$

$r = ?$

By Pythagorus theorem,

$d^2 = r^2 + t^2$

$5^2 = r^2 + 4^2$

$25 = r^2 + 16.$

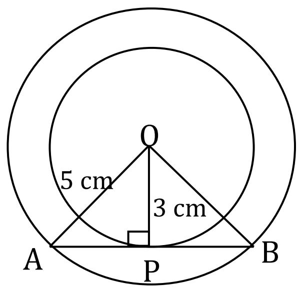
$r^2 = 25 - 16.$

$r = 9$

$r = \sqrt{9}$

$r = 3\ cm.$

6) Two concentric circles are of radii 5 cm and 3 cm . Find the length of the chord of the larger circle which touches the smaller circle.



Solution :

By Pythagorus theorem,

$AO^2 = OP^2 + AP^2$.

$5^2 = 3^2 + AP^2$.

$25 = 9 + AP^2$.

$AP^2 = 25 - 9$.

$AP^2 = 16$.

$AP = \sqrt{16}$

$AP = 4$ cm

$\therefore AB = 8$ cm.

Lesson – 5

Areas Related to Circles

Perimeter :

Perimeter is the length of the boundary of a shape.

Perimeter of a square $= 4 \times side$

Perimeter of a rectangle $= 2(l + b)$

Perimeter of Δ ABC = AB + BC + CA

Perimeter of an equilateral triangle $= 3 \times side$

Circumference of a circle $= 2\pi r$

Area :

Area is the surface covered by a shape.

Area of a square $= side \times side$

Area of a rectangle $= l \times b$

$$Area\ of\ a\ triangle = \frac{1}{2} \times b \times h$$

$$Area\ of\ equilateral\ triangle = \frac{\sqrt{3}.a^2}{4}$$

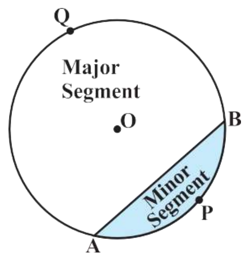
$$Area\ of\ a\ circle = \pi r^2$$

Segment of a circle :

The area bounded by an arc and a chord of a circle is called segment of the circle.

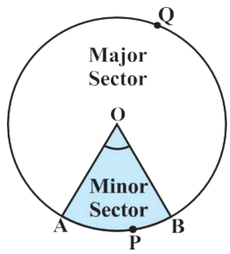
The area bounded by a minor arc and a chord is called minor segment.

The area bounded by a major arc and a chord is called major segment.



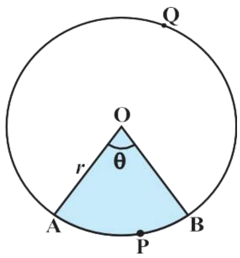
Sector

The area bounded by an arc and two radii of a circle is called a sector.



Area of a sector :

Let OAPB be a sector of a circle with centre O and radius r and of angle θ.



$$Area\ of\ a\ sector = \frac{\theta}{360^0} \times \pi r^2$$

$$Length\ of\ arc\ APB = \frac{\theta}{360^0} \times 2\pi r$$

Problems :

1) If the area of circle is 49π sq.units then find its perimeter.

Solution :

$Area = 49\pi$

$\pi r^2 = 49\pi$

$r^2 = 49$

$r = \sqrt{49}$

$r = 7\ units$

$Perimeter = 2\pi r$

$= 2 \times \frac{22}{7} \times 7$

$= 44\ cm$

2) Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector. (Use $\pi = 3.14$).

Solution :

$$Area\ of\ a\ sector = \frac{\theta}{360^0} \times \pi r^2$$

$$= \frac{30^0}{360^0} \times 3.14 \times 4 \times 4$$

$$= \frac{12.56}{3}$$

$$= 4.19\ cm^2$$

$$Area\ of\ the\ circle = \pi r^2$$

$$= 3.14 \times 4 \times 4$$

$$= 50.24\ cm^2$$

$Area\ of\ the\ corresponding\ major\ sector$

$= 50.24\ cm^2 - 4.19\ cm^2$

$= 46.05\ cm^2$

3) **Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60^0.**

4) **Find the area of a quadrant of a circle whose circumference is 22cm.**

Solution :

$Circumference = 22\ cm$

$2\pi r = 22$

$$r = \frac{22}{2\pi}$$

$$= \frac{22 \times 7}{2 \times 22}$$

$$= 3.5\ cm$$

$$Area\ of\ quadrant = \frac{1}{4} \times \pi r^2$$

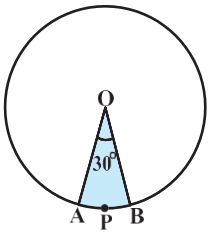
$$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 5.5 \times 0.5 \times 3.5$$

$$= 9.625\ cm^2$$

5) **The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.**

Solution :



Radius of the sector=14 cm

Angle of the sector = 5 minutes

$= 5 \times 6^0$

$= 30^0$

$$Area\ of\ a\ sector = \frac{\theta}{360^0} \times \pi r^2$$

$$= \frac{30^0}{360^0} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{1}{3} \times 11 \times 14$$

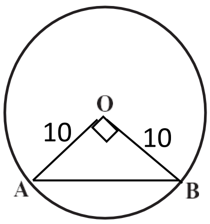
$$= \frac{154}{3}\ cm^2$$

6) **A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding :**

   i. **Minor segment**

   ii. **Major segment.**

Solution :



$$Area\ of\ \Delta\ AOB = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 10 \times 10$$

$$= 50\ cm^2$$

$$Area\ of\ quadrant = \frac{1}{4} \times \pi r^2$$
$$= \frac{1}{4} \times 3.14 \times 10 \times 10$$
$$= \frac{1}{4} \times 3.14 \times 100$$
$$= 3.14 \times 25$$
$$= 78.5\ cm^2$$

$$Area\ of\ circle = \pi r^2$$
$$= 3.14 \times 10 \times 10$$
$$= 3.14 \times 100$$
$$= 314\ cm^2$$

Area of minor segment = 78.5 – 50

= 28.5 cm^2.

Area of major segment = 314 – 28.5

= 285. 5 cm^2.

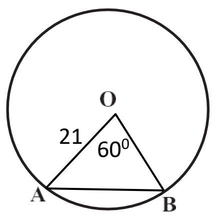
7) In a circle of radius 21 cm, an arc subtends an angle of 60^0 at the centre. Find :

i. The length of the arc.

ii. Area of sector formed by the arc.

iii. Area of the segment formed by the corresponding chord.

Solution :



$$i)\ Length\ of\ the\ arc = \frac{\theta}{360^0} \times 2\pi r$$
$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$$
$$= 22\ cm$$

$$ii)\ Area\ of\ sector = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$$
$$= 231\ cm^2$$

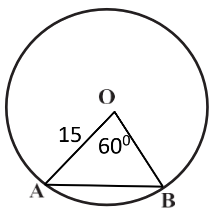
$$iii)\ Area\ of\ \Delta\ AOB = \frac{\sqrt{3}\ a^2}{4}$$
$$= \frac{\sqrt{3} \times 21 \times 21}{4}$$
$$= \frac{441\sqrt{3}}{4}$$

Area of minor segment

$$= \left(231 - \frac{441\sqrt{3}}{4}\right) cm^2$$

8) In a circle of radius 15 cm, an arc subtends an angle of 60^0 at the centre. Find the areas of the corresponding minor and major segments of the circle.

Solution :



$$Area\ of\ sector = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{60}{360} \times 3.14 \times 15 \times 15$$
$$= 1.57 \times 5 \times 15$$
$$= 117.75\ cm^2$$

$$Area\ of\ \Delta\ AOB = \frac{\sqrt{3}\ a^2}{4}$$
$$= \frac{1.7 \times 15 \times 15}{4}$$
$$= \frac{382.5}{4}$$
$$= 95.625\ cm^2$$

$Area\ of\ circle = \pi r^2$

$= 3.14 \times 15 \times 15$

$= 706.5\ cm^2$

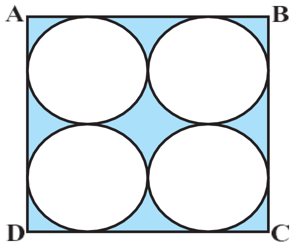
$Area\ of\ minor\ segment = 117.75 - 95.625$

$= 22.125\ cm^2$

$Area\ of\ major\ segment = 706.5 - 22.125$

$= 684.375\ cm^2$

9) Find the area of the shaded region in the figure, where ABCD is a square of side 14 cm.



Solution :

$Area\ of\ square\ = \ side \times side$

$= 14 \times 14$

$= 196\ cm^2$

$Area\ of\ 4\ circles = 4 \times \pi r^2$

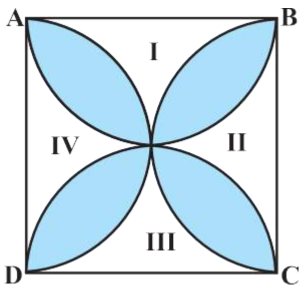
$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 154\ cm^2$

$Area\ of\ shaded\ region = 196 - 154$

$= 42\ cm^2$

10)Find the area of the shaded region in the figure, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter.



Solution :

$Area\ of\ square = side \times side$

$= 10 \times 10$

$= 100\ cm^2$

$Area\ of\ 2\ semicircles = \pi r^2$

$= 3.14 \times 5 \times 5$

$= 78.5\ cm^2$

$Area\ of\ I\ \&\ III = 100 - 78.5$

$= 21.5\ cm^2$

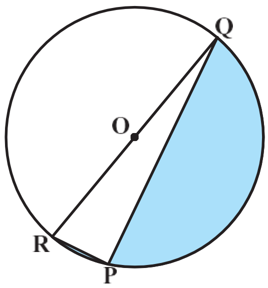
$Area\ of\ II\ \&\ IV = 21.5\ cm^2$

$Area\ of\ I, II, III\ \&\ IV = 43\ cm^2$

$Area\ of\ shaded\ region = 100 - 43$

$= 57\ cm^2$

11)Find the area of the shaded region in the figure, if PQ = 24 cm, PR=7cm and O is the centre of the circle.



Solution :

By Pythagorus theorem,

$QR^2 = \ PR^2 + PQ^2$

$= 7^2 + 24^2$

$= 49 + 576$

$= 625$

$QR = \sqrt{625}$

$= 25\ cm$

$Area\ of\ \Delta\ PQR = \frac{1}{2} \times 7 \times 24$

$= 7 \times 12$

$= 84\ cm^2$

$$Area\ of\ semi\ circle = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$$

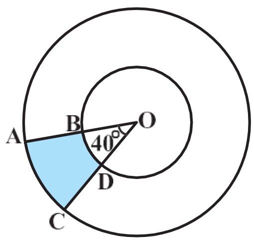
$$= \frac{6875}{28}\ cm^2$$

$$Area\ of\ shaded\ region = \frac{6875}{28} - 84$$

$$= \frac{6875 - 2352}{28}$$

$$= \frac{4523}{28} cm^2$$

12)Find the area of the shaded region in the figure, if radii of two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40^0.



Solution :

$$Area\ of\ sector\ OAC = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{40}{360} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{616}{9}\ cm^2$$

$$Area\ of\ sector\ OBD = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{40}{360} \times \frac{22}{7} \times 7 \times 7$$

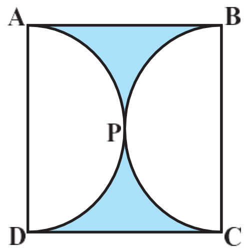
$$= \frac{154}{9}\ cm^2$$

$$Area\ of\ shaded\ region = \frac{616}{9} - \frac{154}{9}$$

$$= \frac{462}{9}$$

$$= \frac{154}{3}\ cm^2$$

13)Find the area of the shaded region in the figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Solution :

$$Area\ of\ square = 14\ cm\ \times 14\ cm$$

$$= 196\ cm^2$$

$$Area\ of\ 2\ semicircles = \pi r^2$$

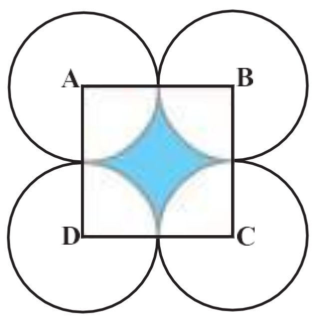
$$= \frac{22}{7} \times 7 \times 7$$

$$= 154\ cm^2$$

$$Area\ of\ shaded\ region = 196 - 154$$

$$= 42\ cm^2$$

14)In the figure, ABCD is a square of side 14 cm. With centers A, B, C and D, four circles are drawn such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



Solution :

$Area\ of\ square = 14\ cm\ \times 14\ cm$

$= 196\ cm^2$

$Area\ of\ 4\ quadrants = \pi r^2$

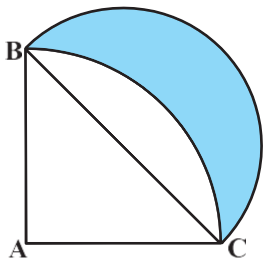
$= \frac{22}{7} \times 7 \times 7$

$= 154\ cm^2$

$Area\ of\ shaded\ region = 196 - 154$

$= 42\ cm^2$

15)In the figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Solution :

$Area\ of\ quadrant = \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154\ cm^2$

$Area\ of\ \Delta\ ABC = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 14 \times 14$

$= 98\ cm^2$

$Area\ of\ segment = 154\ cm^2 - 98\ cm^2$

$= 56\ cm^2$

By Pythagorus theorem,

$BC^2 = AB^2 + AC^2$

$= 14^2 + 14^2$

$= 196 + 196$

$= 2 \times 196$

$BC = \sqrt{2 \times 196}$

$= 14\sqrt{2}$ cm

$Area\ of\ semicircle = \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2}$

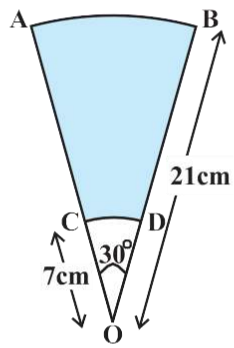
$= 154\ cm^2$

$Area\ of\ shaded\ region$

$= 154\ cm^2 - 56\ cm^2$

$= 98\ cm^2$

16)In the figure, AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If ∠AOB = 30⁰, find the area of the shaded region.



Solution :

$Area\ of\ sector\ OAB = \frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 21 \times 21$

$= \frac{231}{2}\ cm^2$

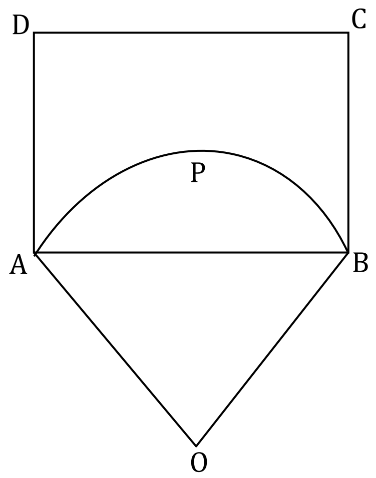
$Area\ of\ sector\ OBD = \frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{77}{6}\ cm^2$

$$Area\ of\ the\ shaded\ region = \frac{231}{2} - \frac{77}{6}$$
$$= \frac{693-77}{6}$$
$$= \frac{616}{6}$$
$$= \frac{308}{3}\ cm^2$$

17)In the figure, ABCD is a rectangle of length 20 cm and breadth 10 cm. OAPB is a sector of a circle of radius $10\sqrt{2}$ cm. Calculate the area of the shaded region. Take $\pi = 3.14$.



Solution ;

$$Area\ of\ quadrant = \frac{1}{4} \times \pi r^2$$
$$= \frac{1}{4} \times 3.14 \times (10\sqrt{2})^2$$
$$= \frac{1}{4} \times 3.14 \times 100 \times 2$$
$$= \frac{1}{2} \times 314$$
$$= 157\ cm^2$$

$$Area\ of\ \Delta\ ABC = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2}$$
$$= \frac{1}{2} \times 100 \times 2$$
$$= 100\ cm^2$$

$$Area\ of\ segment = 157\ cm^2 - 100\ cm^2$$
$$= 57\ cm^2$$

$$Area\ of\ rectangle = 20 \times 10$$
$$= 200\ cm^2$$

Area of shaded region = 200 cm^2 – 57 cm^2

= 143 cm^2.

18)A hand fan is made up of cloth fixed in between the metallic wires. It is in the shape of a sector of a circle of radius 21 cm and of angle 120^0 as shown in the figure. Calculate the area of the cloth used and also find the total length of the metallic wire required to make such a fan.

Solution :

$$Area\ of\ sector = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21$$
$$= 462\ cm^2$$

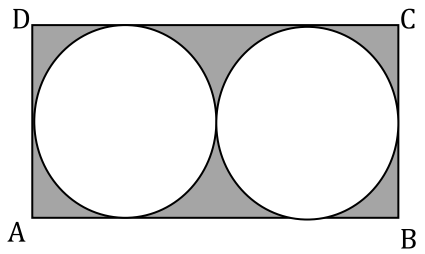
Area of the cloth required is 462 cm^2.

$$Length\ of\ arc = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{120}{360} \times \frac{22}{7} \times 21$$
$$= 22\ cm$$

Length of wire = 21 cm + 21 cm + 22 cm

= 64 cm.

19)In the figure two congruent circles touch each other externally and also touch the sides of the rectangle ABCD. If AB = 28 cm and BC = 14 cm, find the area of the shaded region.



Solution :

$Area\ of\ rectangle = 28 \times 14$

$= 392\ cm^2$

$Area\ of\ 2\ circles = 2\pi r^2$

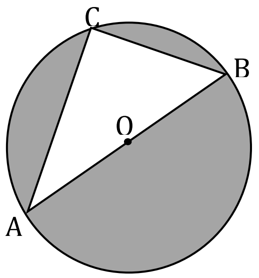
$= 2 \times \frac{22}{7} \times 7 \times 7$

$= 308\ cm^2$

$Area\ of\ shaded\ region = 392\ - 308$

$= 84\ cm^2$

20)A right angled triangle of sides containing right angle are 6 cm and 8 cm, is circumscribed in a circle with centre O or radius 5 cm. Find the area of the shaded region.



Solution :

$Area\ of\ triangle = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times 8$

$= 18\ cm^2$

$Area\ of\ \ circle = \pi r^2$

$= \frac{22}{7} \times 5 \times 5$

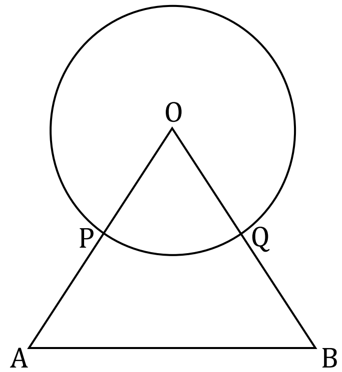
$= 3.14 \times 25$

$= 78.5\ cm^2$

$Area\ of\ shaded\ region = 78.5\ \ - 18\ cm^2$

$= 60.5\ cm^2$

21)In the figure O is the centre of a circle and OAB is an equilateral triangle. P and Q are the midpoints of OA and OB respectively. If the area of Δ OAB is $36\sqrt{3}$ cm², then find the area of the shaded region.



Solution :

$Area\ of\ \Delta\ OAB = 36\sqrt{3}\ cm^2$

$\frac{\sqrt{3}\ a^2}{4} = 36\sqrt{3}$

$\frac{a^2}{4} = 36$

$a^2 = 4 \times 36$

$a = \sqrt{4 \times 36}$

$a = 2 \times 6$

$= 12\ cm$

∴ Radius of the circle = 6 cm

$Area\ of\ \ circle = \pi r^2$

$= \pi \times 6 \times 6$

$= 36\pi\ cm^2$

$Area\ of\ sector\ = \frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times \pi \times 6 \times 6$

$= 6\pi\ cm^2$

Area of shaded region $= 36\,\pi - 6\pi$

$= 30\pi\ cm^2$

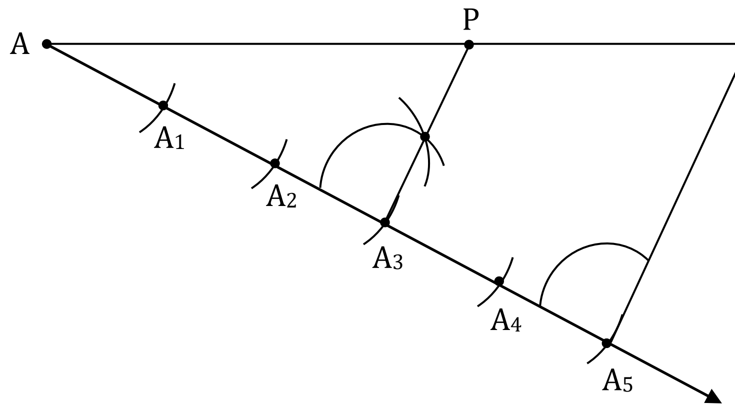
Lesson – 6. Constructions.

Dividing a line segment in the given ratio :

1) Draw a line segment AB = 8 cm and divide it in the ratio 3 : 2.

Solution :

Ratio = 3 : 2



2) Draw a line segment AB = 7.5 cm and divide it in the ratio 4 : 3.

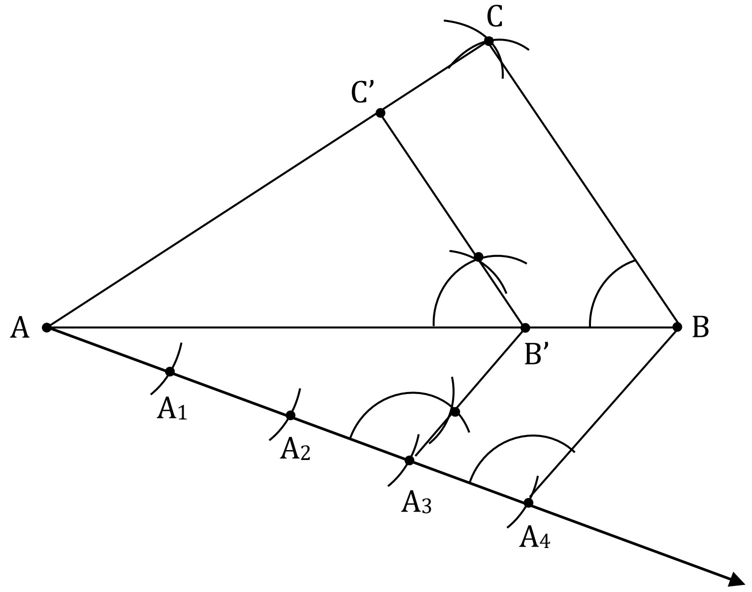
3) Draw a line segment and divide it in the ratio 5 : 8. Measure the two parts.

Construction of a triangle similar to a given triangle as per given scalar factor.

1) Construct a triangle ABC with sides 4 cm, 6 cm and 7 cm and then construct a triangle similar to the given triangle with its sides equal to $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Solution :

$$Ratio = \frac{3}{4}$$

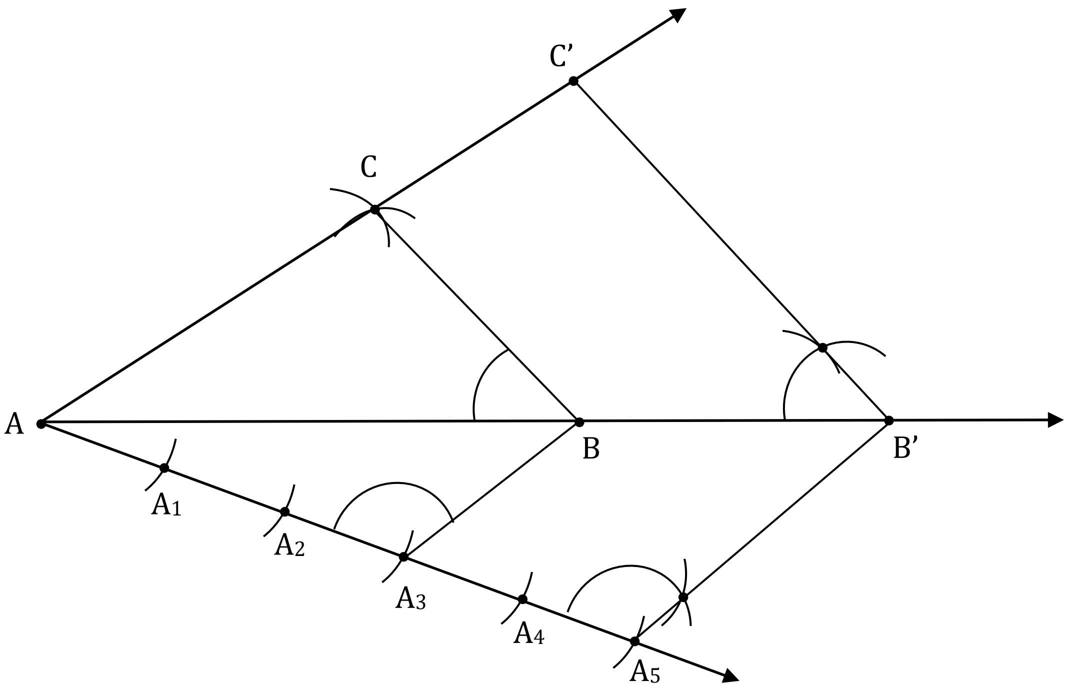


AB'C' is the required triangle.

2) Construct a triangle with sides 3.5 cm, 4.5 cm and 6 cm and then a triangle similar to the given triangle with its sides equal to $\frac{5}{3}$ of the corresponding sides.

Solution :

$$Ratio = \frac{5}{3}$$



AB'C' is the required triangle.

3) Construct a triangle with sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the given triangle.

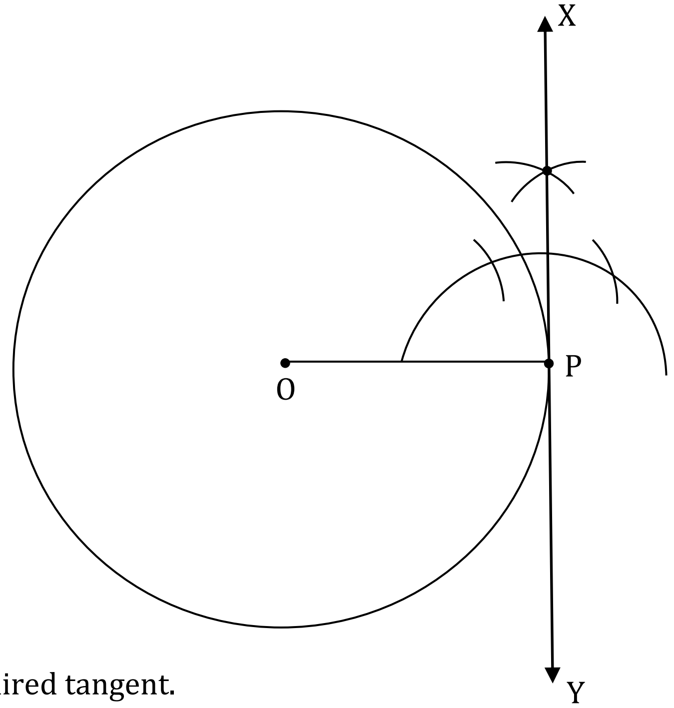
4) Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

5) Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^0$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the given triangle.

Construction of tangents to a circle.

1) Draw a circle of radius 3 cm and construct a tangent at any P on it.

Solution :



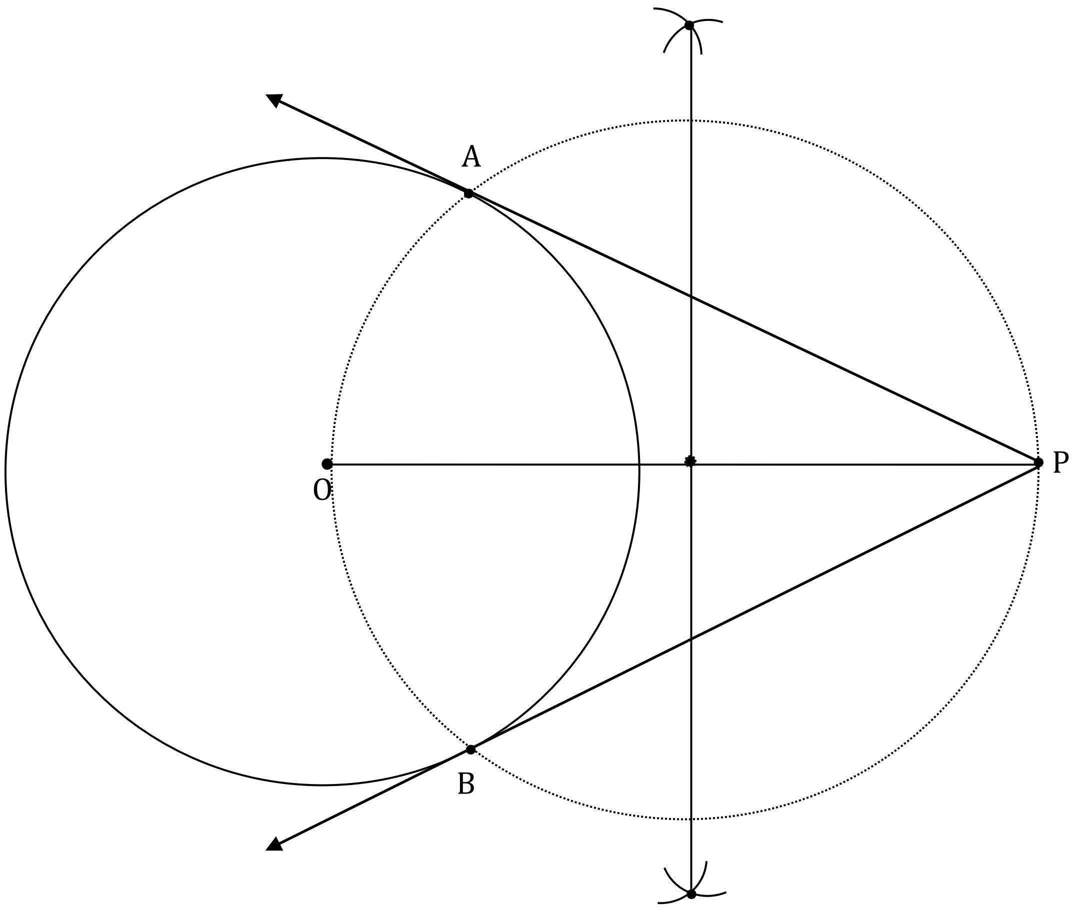
XY is the required tangent.

2) **Draw a circle of radius 4 cm and draw a chord of length 6 cm and construct tangents at the ends of the chord.**

3) **Draw a circle of radius 3.5 cm. From a point 8 cm away from its centre, construct the pair of tangents to the circle.**

Solution :

r = 3.5 cm, d = 8 cm.



PA and PB are required tangents.

4) **Construct a circle of radius 6 cm and then construct two tangents to it from a point 10 cm away from the centre.**

5) **Draw a circle of radius 4 cm. From a point 9 cm away from its centre, construct a pair of tangents to the circle.**

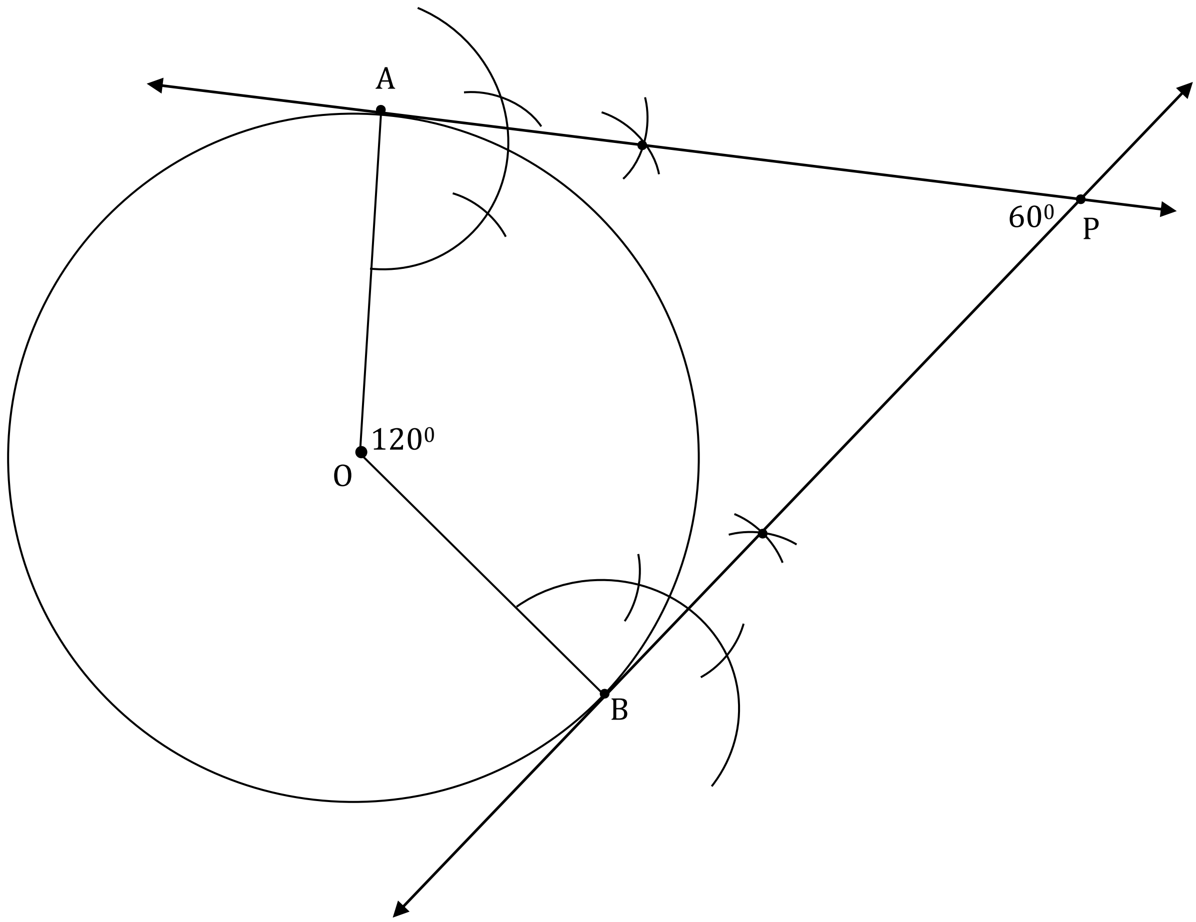
6) **Draw a pair of tangents to a circle of radius 4 cm which are inclined to each other at an angle of 60^0.**

Solution :

Angle between the tangents = 60^0

Angle between the radii = $180^0 - 60^0$

= 120^0



PA and PB are the required tangents.

7) **Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 50^0.**

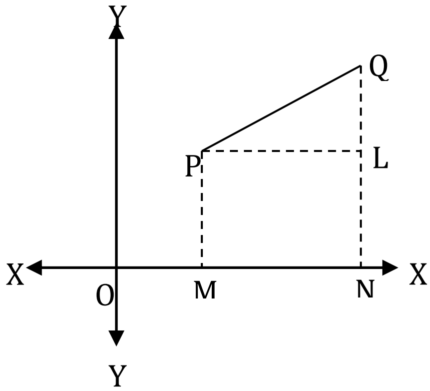
Lesson - 7.

Co-ordinate Geometry.

Distance formula :

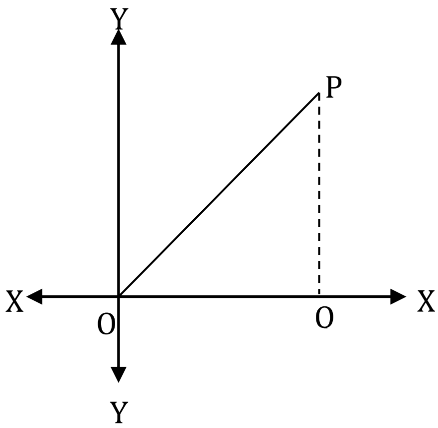
The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the formula,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



The distance of a point P(x, y) from the origin O(0, 0) is given by,

$d = \sqrt{x^2 + y^2}$



Problems :

1) Find the distance between the points A(8, -3) and B(0, 9) using formula.

Solution :

$(x_1, y_1) = (8, -3)$

$(x_2, y_2) = (0, 9)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - 8)^2 + (9 + 3)^2}$

$= \sqrt{(-8)^2 + 12^2}$

$= \sqrt{64 + 144}$

$= \sqrt{208}$

$= \sqrt{16\ X\ 13}$

$= 4\sqrt{13}$ units.

2) Find the distance between the points A(8, 3) and B(2, 11) using formula.

Solution :

$(x_1, y_1) = (8, 3)$

$(x_2, y_2) = (2, 11)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - 8)^2 + (11 - 3)^2}$

$= \sqrt{(-6)^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$=10$ units.

3) Find the distance between the point P(4, 3) and the origin O(0, 0) using formula.

$(x, y) = (4, 3)$

$d = \sqrt{x^2 + y^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$ units.

4) Find the distance of the point (6, 8) from the origin using formula.

$(x, y) = (6, 8)$

$d = \sqrt{x^2 + y^2}$

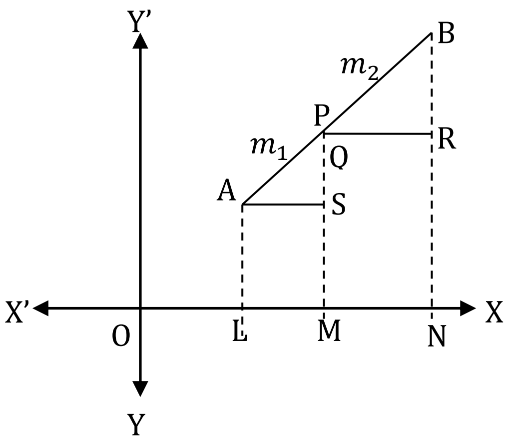
$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$ units.

Section Formula :



The co-ordinates of the point $P(x, y)$ which divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are,

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{my_2 + ny_1}{m_1 + m_2}\right)$$

1) Find the co-ordinates of the point which divides the line segment joining the points (4, -3) and (8, 5) in the ratio 3 : 1 internally.

Solution :

$(x_1, y_1) = (4, -3)$.

$(x_2, y_2) = (8, 5)$.

$m_1 : m_2 = 3 : 1$

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{3(8) + 1(4)}{3 + 1}, \frac{3(5) + 1(-3)}{3 + 1}\right)$$

$$= \left(\frac{24 + 4}{4}, \quad \frac{15 - 3}{4}\right)$$

$$= \left(\frac{28}{4}, \quad \frac{12}{4}\right)$$

$$= (7, 3).$$

2) Find the co-ordinates of the point which divides the line segment joining the points (-1, 7) and (4, -3) in the ratio 2 : 3 internally.

Solution :

$(x_1, y_1) = (-1, 7)$.

$(x_2, y_2) = (4, -3)$.

$m_1 : m_2 = 2 : 3$

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{2(4) + 3(-1)}{2 + 3}, \frac{2(-3) + 3(7)}{2 + 3}\right)$$

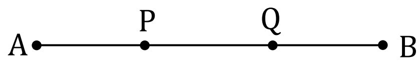
$$= \left(\frac{8 - 3}{5}, \quad \frac{-6 + 21}{5}\right)$$

$$= \left(\frac{5}{5}, \quad \frac{15}{5}\right)$$

$$= (1, 3).$$

3) Find the co-ordinates of the points of trisection of the line segment joining the points (2, -2) and (-7, 4).

Solution :



$(x_1, y_1) = (2, -2)$.

$(x_2, y_2) = (-7, 4)$.

$m_1 : m_2 = 1 : 2$

$$P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{1(-7) + 2(2)}{1 + 2}, \frac{1(4) + 2(-2)}{1 + 2}\right)$$

$$= \left(\frac{-7 + 4}{3}, \quad \frac{4 - 4}{3}\right)$$

$$= \left(\frac{-3}{3}, \quad \frac{0}{3}\right)$$

$$= (-1, \ 0).$$

$(x_1, y_1) = (2, -2)$.

$(x_2, y_2) = (-7, 4)$.

$m_1 : m_2 = 2 : 1$

$$Q = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{2(-7) + 1(2)}{1 + 2}, \frac{2(4) + 1(-2)}{1 + 2}\right)$$

$$= \left(\frac{-14+2}{3}, \quad \frac{8-2}{3}\right)$$

$$= \left(\frac{-12}{3}, \quad \frac{6}{3}\right)$$

$$= (-4, \ 2).$$

4) In what ratio does the point (- 4, 6) divide the line segment joining the points A(- 6, 10) and B(3, - 8)?

Solution :

$P(x, y) = (-4, 6)$

$(x_1, y_1) = (-6, 10).$

$(x_2, y_2) = (3, -8).$

$m_1 : m_2 = ?$

$x - coordinate = -4$

$$\frac{m_1x_2 + m_2x_1}{m_1 + m_2} = -4$$

$$\frac{m_1(3) + m_2(-6)}{m_1 + m_2} = -4$$

$3m_1 - 6m_2 = -4m_1 - 4m_2$

$3m_1 + 4m_1 = -4m_2 + 6m_2$

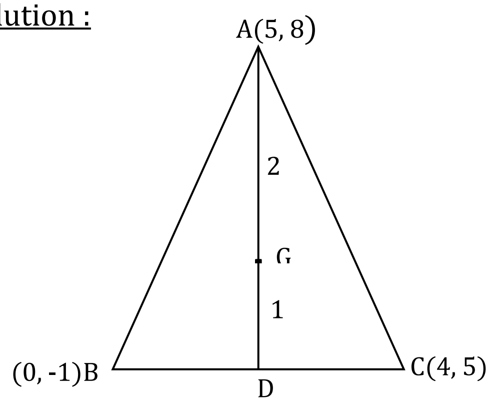
$7m_1 = 2m_2$

$$\frac{m_1}{m_2} = \frac{2}{7}$$

$m_1 : m_2 = 2 : 7$

5) A(5, 8), B(0, -1) and C(4, 5) are the vertices of a Δ ABC. AD is the median and G is a point on AD such that AG : GD = 2 : 1. Find the co-ordinate of the point G.

Solution :



$$\therefore Coordinates\ of\ D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{0+4}{2}, \frac{-1+5}{2}\right)$$

$$= \left(\frac{4}{2}, \frac{4}{2}\right)$$

$$= (2, 2)$$

$(x_1, y_1) = (5, 8).$

$(x_2, y_2) = (2, 2).$

$m_1 : m_2 = 2 : 1$

$$G = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{2(2) + 1(5)}{2+1}, \frac{2(2) + 1(8)}{2+1}\right)$$

$$= \left(\frac{4+5}{3}, \quad \frac{4+8}{3}\right)$$

$$= \left(\frac{9}{3}, \quad \frac{12}{3}\right)$$

$$= (3, \ 4).$$

Mid-point formula :

The co-ordinates of the mid-point $P(x, y)$ of the line joining the points A(x_1, y_1) and B(x_2, y_2) are,

$$P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

6) Find the co-ordinates of the mid-point of the line segment joining the points (-2, 7) and (4, -3) .

Solution :

$(x_1, y_1) = (-2, 7).$

$(x_2, y_2) = (4, -3).$

$$P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-2+4)}{2}, \frac{7-3)}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{4}{2}\right)$$

$$= (1, 2)$$

7) Find the co-ordinates of the mid-point of the line segment joining the points (5, 9) and (3, 1).

Solution :

$(x_1, y_1) = (5, 9)$.

$(x_2, y_2) = (3, 1)$.

$$P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

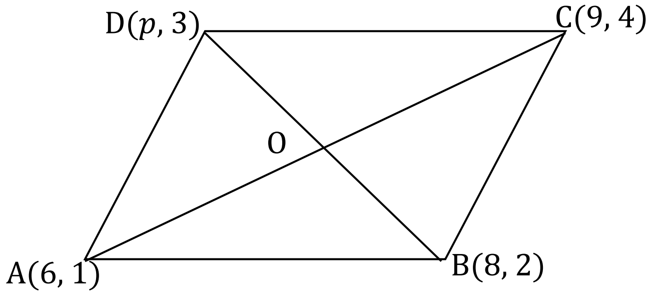
$$= \left(\frac{5+3)}{2}, \frac{9+1)}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{10}{2}\right)$$

$$= (4, 5)$$

8) If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram taken in order, find the value of p.

Solution :



Diagonals bisect each other.

$\therefore$ *Midpoint of AC = Midpoint of DB*

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{p+8}{2}, \frac{3+2}{2}\right)$$

$$\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{p+8}{2}, \frac{5}{2}\right)$$

Comparing x-coordinates, we get

$$\frac{p+8}{2} = \frac{15}{2}$$

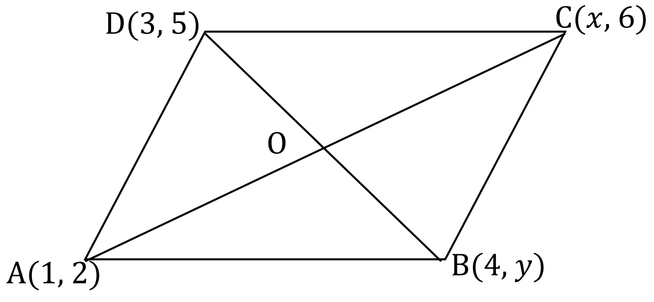
$p + 8 = 15$

$p = 15 - 8.$

$p = 7.$

9) If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Solution :



Diagonals of a parallelogram bisect each other.

$\therefore$ *Midpoint of AC = Midpoint of DB*

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{5+y}{2}\right)$$

$$\left(\frac{1+x}{2}, \frac{8}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Comparing x-coordinates, we get

$$\frac{1+x}{2} = \frac{7}{2}$$

$1 + x = 7$

$x = 7 - 1.$

$x = 6.$

Comparing y-coordinates, we get

$$\frac{5+y}{2} = \frac{8}{2}$$

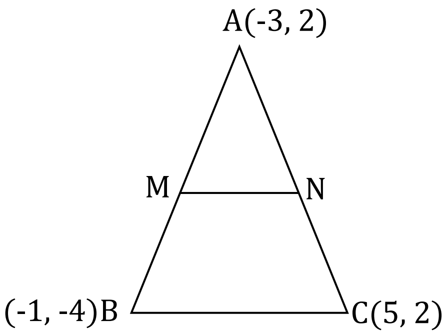
$5 + y = 8$

$y = 8 - 5.$

$y = 3.$

10) The vertices of a Δ ABC are A(-3, 2), B(-1, -4) and C(5, 2). If M and N are the midpoints of AB and AC respectively, show that 2MN = BC.

Solution :



$Coordinates\ of\ M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-3 - 1}{2}, \frac{2 - 4}{2}\right)$

$= \left(\frac{-4}{2}, \frac{-2}{2}\right)$

= (-2, -1)

$Coordinates\ of\ N = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-3 + 5}{2}, \frac{2 + 2}{2}\right)$

$= \left(\frac{2}{2}, \frac{4}{2}\right)$

= (1, 2)

$B(-1, -4) = (x_1, y_1)$.

$C(5,\ 2) = (x_2\ , y_2)$.

$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5 + 1)^2 + (2 + 4)^2}$

$= \sqrt{(6)^2 + (6)^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$

$= \sqrt{36\ X\ 2}$

$= 6\sqrt{2}$ units.

$M(-2, -1) = (x_1, y_1)$.

$N(1,\ 2) = (x_2\ , y_2)$.

$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 + 2)^2 + (2 + 1)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18}$

$= \sqrt{9\ X\ 2}$

$= 3\sqrt{2}$ units.

∴ BC = 2MN.

Hence proved.

Area of a triangle ABC with given vertices.

Area of a triangle ABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by,

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

1) Find the area of the triangle ABC whose vertices are A(2, 3), B(-1, 0) and C(2, -4).

Solution :

$A(2,3) = (x_1, y_1)$

$B(-1, 0) = (x_2, y_2)$

$C(2, -4) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[2(0 + 4) - 1(-4 - 3) + 2(3 - 0)]$

$= \frac{1}{2}[2(4) - 1(-7) + 2(3)]$

$= \frac{1}{2}[8 + 7 + 6]$

$= \frac{1}{2}(21)$

$= \frac{21}{2}\ square\ units.$

2) Find the area of the triangle ABC whose vertices are A(1, -1), B(-4, 6) and C(-3, -5).

Solution :

$A(1,-1) = (x_1, y_1)$

$B(-4,6) = (x_2, y_2)$

$C(-3,-5) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$

$= \frac{1}{2}[1(6+5) - 4(-5+1) - 3(-1-6)]$

$= \frac{1}{2}[1(11) - 4(-4) - 3(-7)]$

$= \frac{1}{2}[11 + 16 + 21]$

$= \frac{1}{2}(48)$

$= 24\ square\ units.$

3) Find the area of the triangle ABC whose vertices are A(5, 2), B(4, 7) and C(7, -4).

Solution :

$A(5,2) = (x_1, y_1)$

$B(4,7) = (x_2, y_2)$

$C(7,-4) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[5(7+4) + 4(-4-2) + 7(2-7)]$

$= \frac{1}{2}[5(11) + 4(-6) + 7(-5)]$

$= \frac{1}{2}[55 - 24 - 35]$

$= \frac{1}{2}(55 - 59)$

$= \frac{1}{2}(-4)$

$= (-2)$

$= 2\ square\ units.$

4) Find the area of the triangle with vertices are P(-1.5, 3), Q(6, 2) and R(-3, 4).

Solution :

$A(-1.5, 3) = (x_1, y_1)$

$B(6,\ 2) = (x_2, y_2)$

$C(-3,4) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-1.5(2-4) + 6(4-3) - 3(3-2)]$

$= \frac{1}{2}[-1.5(-2) + 6(1) - 3(1)]$

$= \frac{1}{2}[3 + 6 - 3]$

$= \frac{1}{2}(6)$

$= 3$ square units

5) Show that the points (7, -2), (5, 1) and (3, 4) are collinear.

$A(7,-2) = (x_1, y_1)$

$B(5,1) = (x_2, y_2)$

$C(3,4) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[7(1-4) + 5(4+2) + 3(-2-1)]$

$= \frac{1}{2}[7(-3) + 5(6) + 3(-3)]$

$= \frac{1}{2}[-21 + 30 - 9]$

$= \frac{1}{2}(30 - 30)$

$= \frac{1}{2}(0)$

$= 0.$

Area is zero.

∴ The given points are collinear.

6) Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

Solution :

$A(1,5) = (x_1, y_1)$

$B(2,3) = (x_2, y_2)$

$C(-2,-11) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(3 + 11) + 2(-11 - 5) - 2(5 - 3)]$

$= \frac{1}{2}[1(14) + 2(-16) - 2(2)]$

$= \frac{1}{2}[14 - 32 - 4]$

$= \frac{1}{2}(14 - 36)$

$= \frac{1}{2}(-22)$

= (-11)

= 11 square units.

Area is not zero.

∴ The given points are not collinear.

7) Find the value of k if the points A(2, 3), B(4, k) and C(6, -3) are collinear.

Solution :

$A(2,3) = (x_1, y_1)$

$B(4,k) = (x_2, y_2)$

$C(6,-3) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[2(k + 3) + 4(-3 - 3) + 6(3 - k)]$

$= \frac{1}{2}[2k + 6 + 4(-6) + 18 - 6k]$

$= \frac{1}{2}[2k + 6 - 24 + 18 - 6k]$

$= \frac{1}{2}(-4k - 18 + 18)$

$= \frac{1}{2}(-4k)$

= 2k.

The points are collinear.

$\therefore\ Area\ =\ 0.$

$2k\ =\ 0$

$k\ = \frac{0}{2}$

$k = 0$

8) Find the value of k if the points A(7, -2), B(5, 1) and C(3, k) are collinear.

Solution :

$A(7,-2) = (x_1, y_1)$

$B(5,1) = (x_2, y_2)$

$C(3,k) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[7(1 - k) + 5(k + 2) + 3(-2 - 1)]$

$= \frac{1}{2}[7 - 7k + 5k + 10 + 3(-3)]$

$= \frac{1}{2}[7 - 2k + 10 - 9]$

$= \frac{1}{2}(7 - 2k + 1)$

$= \frac{1}{2}(8 - 2k)$

The points are collinear.

∴ Area = 0.

$\frac{1}{2}(8 - 2k) = 0$

$8 - 2k = 0$

$2k = 8$

$k = \frac{8}{2}$

$k\ =\ 4.$

9) Find the value of k if the points A(8, 1), B(k,-4) and C(2, -5) are collinear.

Solution :

$A(8,1) = (x_1, y_1)$

$B(k,-4) = (x_2, y_2)$

$C(2,-5) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[8(-4+5) + k(-5-1) + 2(1+4)]$

$= \frac{1}{2}[8(1) + k(-6) + 2(5)]$

$= \frac{1}{2}[8 - 6k + 10]$

$= \frac{1}{2}(18 - 6k)$

The points are collinear.

$\therefore$ Area = 0.

$\frac{1}{2}(18 - 6k) = 0$

$18 - 6k = 0$

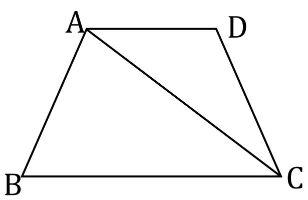
$6k = 18$

$k = \frac{18}{6}$

$k = 3.$

10)If A(-4, 5), B(0, 7), C(5, -5) and D(-4, 2) are the vertices of a quadrilateral ABCD, find the area of the quadrilateral.

Solution :



$A(-4,5) = (x_1, y_1)$

$B(0,7) = (x_2, y_2)$

$C(5,-5) = (x_3, y_3)$

$Area\ of\ \Delta\ ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-4(7+5) + 0 + 5(5-7)]$

$= \frac{1}{2}[-4(12) + 5(-2)]$

$= \frac{1}{2}[-48 - 10]$

$= \frac{1}{2}(-58)$

= -29

= 29 square units.

$A(-4,5) = (x_1, y_1)$

$D(-4,2) = (x_2, y_2)$

$C(5,-5) = (x_3, y_3)$

$Area\ of\ \Delta\ ADC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-4(7+5) - 4(-5-5) + 5(5-2)]$

$= \frac{1}{2}[-4(12) - 4(-10) + 5(3)]$

$= \frac{1}{2}(-48 + 40 + 15)$

$= \frac{1}{2}(-8 + 15)$

$= \frac{1}{2}(7)$

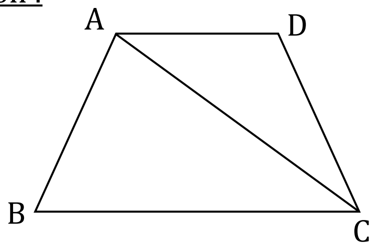
$= 3.5\ square\ units.$

Area of quadrilateral = 29 + 3.5

= 32.5 square units.

11)Find the area of the quadrilateral whose vertices taken in order are (-4, -2), (-3, -5), (3, -2) and (2, 3).

Solution :



$A(-4,-2) = (x_1, y_1)$

$B(-3,-5) = (x_2, y_2)$

$C(3,-2) = (x_3, y_3)$

Area of $\Delta\, ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-4(-5+2) - 3(-2+2) + 3(-2+5)]$

$= \frac{1}{2}[-4(-3) - 3(0) + 3(3)]$

$= \frac{1}{2}[12 - 0 + 9]$

$= \frac{1}{2}[21]$

$= 10.5$ square units.

$A(-4,-2) = (x_1, y_1)$

$D(2,3) = (x_2, y_2)$

$C(3,-2) = (x_3, y_3)$

Area of $\Delta\, ADC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-4(3+2) + 2(-2+2) + 3(-2-3)]$

$= \frac{1}{2}[-4(5) + 2(0) + 3(-5)]$

$= \frac{1}{2}[-20 - 0 - 15]$

$= \frac{1}{2}(-35)$

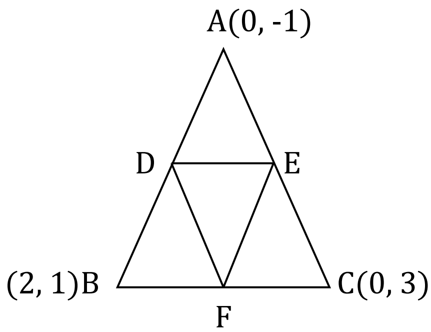
$= 17.5$ square units.

Area of quadrilateral $= 10.5 + 17.5$

$= 28$ square units.

12)Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of this triangle.

Solution :



$Co-ordinates\ of\ D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{2+0}{2}, \frac{1-1}{2}\right)$

$= \left(\frac{2}{2}, \frac{0}{2}\right)$

$= (1, 0)$

$Co\ ordinates\ of\ E = \left(\frac{0+0}{2}, \frac{-1+3}{2}\right)$

$= \left(\frac{0}{2}, \frac{2}{2}\right)$

$= (0, 1)$

$Co\ ordinates\ of\ F = \left(\frac{2+0}{2}, \frac{1+3}{2}\right)$

$= \left(\frac{2}{2}, \frac{4}{2}\right)$

$= (1, 2)$

D$(1, 0) = (x_1, y_1)$

E$(0, 1) = (x_2, y_2)$

F$(1, 2) = (x_3, y_3)$

Area of ΔDEF

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(1-2) + 0 + 1(0-1)]$

$= \frac{1}{2}[1(-1) + 1(-1)]$

$= \frac{1}{2}(-1-1)$

$= \frac{1}{2}(-2)$

$= -1$

$= 1$ *square units.*

A(0, -1) $= (x_1, y_1)$

B(2, 1) $= (x_2, y_2)$

C(0, 3) $= (x_3, y_3)$

Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[0 + 2(3+1) + 0]$

$= \frac{1}{2}[2(4)]$

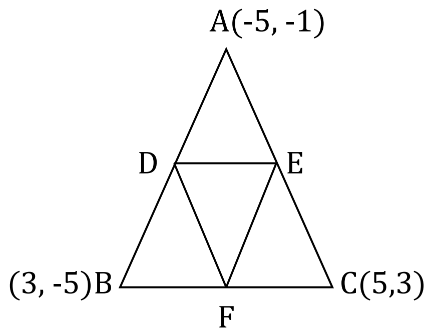
$= \frac{1}{2}(8)$

= 4 square units.

$\therefore \frac{\Delta ABC}{\Delta DEF} = \frac{4}{1}$

13)The vertices of Δ ABC are A(-5, -1), B(3, -5) and C(5, 3). Show that the area of Δ ABC is 4 times the area o triangle formed by joining the mid-points of the sides of the triangle.

Solution :



$Co-ordinates\ of\ D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{3-5}{2}, \frac{-5-1}{2}\right)$

$= \left(\frac{-2}{2}, \frac{-6}{2}\right)$

= (-1, -3)

$Co\ ordinates\ of\ E = \left(\frac{-5+5}{2}, \frac{-1+3}{2}\right)$

$= \left(\frac{0}{2}, \frac{2}{2}\right)$

=(0, 1)

$Co\ ordinates\ of\ F = \left(\frac{3+5}{2}, \frac{-5+3}{2}\right)$

$= \left(\frac{8}{2}, \frac{-2}{2}\right)$

=(4, -1)

D(-1, -3) $= (x_1, y_1)$

E(0, 1) $= (x_2, y_2)$

F(4, -1) $= (x_3, y_3)$

Area of ΔDEF

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-1(1+1) + 0 + 4(-3-1)]$

$= \frac{1}{2}[-1(2) + 4(-4)]$

$= \frac{1}{2}(-2-16)$

$= \frac{1}{2}(-18)$

$= -9$

$= 9$ *square units.*

A(-5, -1) = (x_1, y_1)

B(3, -5) = (x_2, y_2)

C(5, 3) = (x_3, y_3)

$Area\ of \Delta ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5(-5-3) + 3(3+1) + 5(-1+5)]$

$= \frac{1}{2}[-5(-8) + 3(4) + 5(4)]$

$= \frac{1}{2}(40 + 12 + 20)$

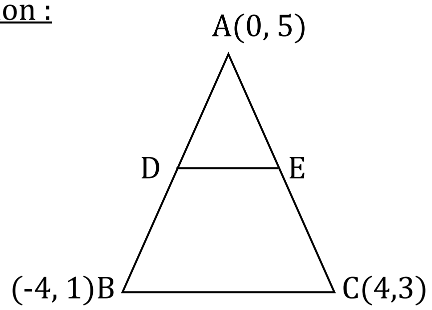
$= \frac{1}{2}(72)$

$= 36\ square\ units.$

$\therefore\ ar(\Delta ABC) = 4 \times ar(\Delta DEF)$

14)In Δ ABC with vertices A(0, 5), B(-4, 1) and C(4, 3), D and E are the mid-points of the sides AB and AC respectively, then find the area of Δ ADE.

Solution :



$Co - ordinates\ of\ D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-4+0}{2}, \frac{1+5}{2}\right)$

$= \left(\frac{-4}{2}, \frac{6}{2}\right)$

= (-2, 3)

$Co\ ordinates\ of\ E = \left(\frac{0+4}{2}, \frac{5+3}{2}\right)$

$= \left(\frac{4}{2}, \frac{8}{2}\right)$

=(2, 4)

A(0, 5) = (x_1, y_1)

D(-2, 3) = (x_2, y_2)

E(2, 4) = (x_3, y_3)

$Area\ of\ \Delta ADE$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[0 - 2(4-5) + 2(5-3)]$

$= \frac{1}{2}[-2(-1) + 2(2)]$

$= \frac{1}{2}(2+2)$

$= \frac{1}{2}(4)$

= 2 square units.

Lesson – 8. Real Numbers

Euclid's Division Lemma :

"Given positive integers a and b,there exist unique integers q and r satisfying $a = bq + r, 0 \leq r < b$". This is called Euclid's division lemma.

Fundamental theorem of Arithmetic :

"Every composite number can be expressed as a product of its prime factors". This is called Fundamental theorem of Arithemetic.

For any two positive integers a and b,

$HCF(a, b) \times LCM(a, b) = a \times b$

Rational numbers :

A number which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.

Irrantional Numbers :

A number which cannot be expressed in the form $\frac{p}{q}$, where p, q are integers & $q \neq 0$, is called an irrational number.

Examples : $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}$

Real numbers :

Rational numbers and irrational numbers are together called real numbers.

Let p is a prime number and a is a positive integer. If p divides a^2, then p divides a.

If a rational number has a terminating decimal expansion, then its denominator has the prime factorization of the form 2^m5^n.

If a rational number has a non-terminating or repeating or recurring decimal expansion, then its denominator does not have the prime factorization of the form 2^m5^n.

HCF and LCM.

HCF of two positive integers a and b is the greatest number that divides a and b.

HCF of two co-prime numbers is 1.

LCM of two positive integers a and b is the least number that is divisible by a and b.

LCM of to co-prime numbers is equal to their product.

$$HCF(a, b) \times LCM(a, b) = a \times b$$

Problems :

1) Find the HCF of 24 and 40 by using Euclid's division algorithm. Hence find the LCM of HCF(24, 40) and 20.

Solution :

24) 40 (1
24
16

16) 24 (1
16
8

8) 16 (2
16
0

HCF(24, 40) = 8

8) 20 (2
16
4

4) 8 (2
8
0

HCF(8,20) = 4

$$LCM(8, 20) = \frac{a \times b}{H}$$
$$= \frac{8 \times 20}{4}$$
$$= 40$$

2) Find the HCF of 135 and 75 by the prime factorization, hence find the LCM of HCF(135, 75) and 20.

Solution :

75) 135 (1
75
60

60) 75 (1
60
15

15) 60 (4
60
0

HCF(135, 75) = 15

15) 20 (1
15
5

5) 15 (3
15
0

HCF(15, 20) = 5

$$LCM(15, 20) = \frac{a \times b}{H}$$
$$= \frac{15 \times 20}{5}$$
$$= 60$$

3) Express 140 as a product of its prime factors.

Solution :

5	140
2	28
2	14
	7

$140 = 5 \times 2^2 \times 7$

4) Express 156 as a product of its prime factors.

Solution :

2	156
2	78
3	39
	13

$156 = 3 \times 2^2 \times 13$

5) Find the HCF of 96 and 404 by the prime factorization method. Hence find their LCM.

Solution :

2	96
2	48
2	24
2	12
2	6
	3

2	404
2	202
	101

$96 = 2^5 \times 3$

$404 = 2^2 \times 101$

$HCF = 2^2 = 4$

$$LCM = \frac{a \times b}{H}$$
$$= \frac{96 \times 404}{4}$$
$$= 24 \times 404$$
$$= 9696$$

6) Find the HCF and LCM of 6, 72 and 120 using prime factorization method.

Solution :

5	120
2	24
2	12
2	6
	3

2	6
	3

2	72
2	36
2	18
3	9
	3

$120 = 2^3 \times 3 \times 5$

$72 = 2^3 \times 3^2$

$6 = 2 \times 3$

$HCF = 2 \times 3$

$= 6$

$LCM = 2^3 \times 3^2 \times 5$

$= 8 \times 9 \times 5$

$= 360$

7) Express the denominator of $\frac{23}{20}$ in in the form of $2^n \times 5^m$.

Solution :

5	20
2	4
	2

$20 = 5^1 \times 2^2$

It is terminating decimal.

8) Prove that $7 \times 11 \times 13 + 13$ is a composite number.

Solution :

$7 \times 11 \times 13 + 13$

$= 77 \times 13 + 13$

$= 13(77 + 1)$

$= 13 \times 78$

It is expressed as a product of prime factors.

$\therefore$ It is a composite number.

9) Prove that $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number.

Solution :

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 1008 \times 5 + 5$

$= 5(1008 + 1)$

$= 5 \times 1009$

It is expressed as a product of prime factors.

$\therefore$ It is a composite number.

10) Show that $\sqrt{2}$ is irrational.

Solution :

Assume that $\sqrt{2}$ *is rational.*

$Let\ \sqrt{2} = \frac{a}{b}$, $a\ and\ b\ are\ co-prime.$

$\Rightarrow \sqrt{2}b = a$

$\Longrightarrow (\sqrt{2}b)^2 = a^2$

$\Longrightarrow 2b^2 = a^2$

$\Longrightarrow 2\ divides\ a^2$

$\Longrightarrow 2\ divides\ a$

$\Longrightarrow a = 2c\ for\ some\ integer\ c.$

$\Longrightarrow a^2 = 4c^2$

$\Longrightarrow 2b^2 = 4c^2$

$\Longrightarrow b^2 = 2c^2$

$\Longrightarrow 2\ divides\ b^2$

$\Longrightarrow 2\ divides\ b$

$\Longrightarrow 2\ divides\ both\ a\ and\ b$

$\Longrightarrow a \text{ and } b \text{ are not coprime}$

This is a contradiction.

$\therefore \sqrt{2} \text{ is irrational.}$

11)Show that $\sqrt{3}$ is irrational.

Solution :

Assume that $\sqrt{3}$ *is rational.*

$Let\ \sqrt{3} = \frac{a}{b}\ ,\ \ a \text{ and } b \text{ are co} - prime.$

$\Rightarrow \sqrt{3}b = a$

$\Longrightarrow (\sqrt{3}b)^2 = a^2$

$\Longrightarrow 3b^2 = a^2$

$\Longrightarrow 3 \text{ divides } a^2$

$\Longrightarrow 3 \text{ divides } a$

$\Longrightarrow a = 3c \text{ for some integer } c.$

$\Longrightarrow a^2 = 9c^2$

$\Longrightarrow 3b^2 = 9c^2$

$\Longrightarrow b^2 = 3c^2$

$\Longrightarrow 3 \text{ divides } b^2$

$\Longrightarrow 3 \text{ divides } b$

$\Longrightarrow 3 \text{ divides both } a \text{ and } b$

$\Longrightarrow a \text{ and } b \text{ are not coprime}$

This is a contradiction.

$\therefore \sqrt{3}$ is irrational.

12) Show that $\sqrt{5}$ is irrational.

Solution :

Assume that $\sqrt{5}$ *is rational.*

$Let\ \sqrt{5} = \frac{a}{b}\ ,\ \ a \text{ and } b \text{ are co} - prime.$

$\Rightarrow \sqrt{5}b = a$

$\Longrightarrow (\sqrt{5}b)^2 = a^2$

$\Longrightarrow 5b^2 = a^2$

$\Longrightarrow 5 \text{ divides } a^2$

$\Longrightarrow 5 \text{ divides } a$

$\Longrightarrow a = 5c \text{ for some integer } c.$

$\Longrightarrow a^2 = 25c^2$

$\Longrightarrow 5b^2 = 25c^2$

$\Longrightarrow b^2 = 5c^2$

$\Longrightarrow 5 \text{ divides } b^2$

$\Longrightarrow 5 \text{ divides } b$

$\Longrightarrow 5 \text{ divides both } a \text{ and } b$

$\Longrightarrow a \text{ and } b \text{ are not coprime}$

This is a contradiction.

$\therefore \sqrt{5} \text{ is irrational.}$

13)Show that $3 + \sqrt{5}$ is irrational.

Solution :

$Assume\ that\ \ 3 + \sqrt{5} \text{ is rational.}$

$Let\ 3 + \sqrt{5} = \frac{a}{b}, a \text{ and } b \text{ are coprime.}$

$\Longrightarrow \sqrt{5} = \frac{a}{b} - 3$

$\Longrightarrow \sqrt{5} = \left(\frac{a - 3b}{b}\right)$

$\Longrightarrow \sqrt{5} \text{ is rational}$

This is a contradiction.

$\therefore 3 + \sqrt{5} \text{ is irrational}$

14)Show that 6+$\sqrt{2}$ is irrational.

Solution :

$Assume\ that\ \ 6 + \sqrt{2} \text{ is rational.}$

$Let\ 6 + \sqrt{2} = \frac{a}{b}, a \text{ and } b \text{ are coprime.}$

$\Longrightarrow \sqrt{2} = \frac{a}{b} - 6$

$\Longrightarrow \sqrt{2} = \left(\frac{a - 6b}{b}\right)$

$\Longrightarrow 2 \text{ is rational}$

This is a contradiction.

$\therefore 6 + \sqrt{2} \text{ is irrational}$

15)Show that $3 + 2\sqrt{5}$ is irrational.

Solution :

$Assume\ that\ \ 3 + 2\sqrt{5} \text{ is rational.}$

$Let\ 3 + 2\sqrt{5} = \frac{a}{b}, a\ and\ b\ are\ coprime.$

$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$

$\Rightarrow 2\sqrt{5} = \left(\frac{a - 3b}{b}\right)$

$\Rightarrow \sqrt{5} = \left(\frac{a - 3b}{2b}\right)$

$\Rightarrow \sqrt{5}\ is\ rational$

This is a contradiction.

$\therefore 3 + 2\sqrt{5}\ is\ irrational$

16)Show that $5 - \sqrt{3}$ is irrational.

Solution :

$Assume\ that\ \ 5 - \sqrt{3}\ is\ rational.$

$Let\ 5 - \sqrt{3} = \frac{a}{b}, a\ and\ b\ are\ coprime$

$\Rightarrow \sqrt{3} = 5 - \frac{a}{b}$

$\Rightarrow \sqrt{3} = \left(\frac{5b - a}{b}\right)$

$\Rightarrow \sqrt{3}\ is\ rational$

This is a contradiction.

$\therefore 5 - \sqrt{3}\ is\ irrational$

Lesson – 9. Polynomials

A polynomial is an algebraic expression having only positive integers as exponents.

Standard form of a polynomial :

The standard form or general form of a polynomial is,

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots a_2 x^2 + a_1 x + a_0.$

where x is the variable and $a_0, a_1, a_2 \ldots .$ are constants and $a_n \neq 0$.

Degree of a polynomial :

The highest power of the variable in the polynomial is called degree of the polynomial.

A polynomial of degree 1 is called a linear polynomial and its standard form is $p(x) = ax + b$.

A polynomial of degree 2 is called a quadratic polynomial and its standard form is given by, $p(x) = ax^2 + bx + c$.

A polynomial of degree 3 is called a cubic polynomial and its standard form is given by, $p(x) = ax^3 + bx^2 + cx + d$.

Zero of a polynomial :

A zero of a polynomial $p(x)$ is a real number 'c' such that p(c) = 0. The number 'c' is called the root of the equation p(x) = 0.

A linear polynomial has one zero.

A quadratic polynomial has two zeroes.

A cubic polynomial has three zeroes.

Graphs of polynomials :

The graph of the equation $y = ax + b$ is a straight line.

Graph of the equation $y = ax^2 + bx + c$ is a parabola.

Division Algorithm :

If a polynomial $p(x)$ is divided by an another polynomial $g(x)$, then

$p(x) = g(x).q(x) + r(x)$.

Where, $q(x)$ is the quotient and $r(x)$ is the remainder.

That is,

Dividend = Divisor × Quotient + Remainder

Relationship between zeroes and coefficients :

Let α and β are two zeroes of the quadratic polynomial $P(x) = ax^2 + bx + c$, then sum and product of zeroes are given by,

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Let α, β and γ are three zeroes of the cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$, then,

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

Problems :

1) If $P(x) = x^2 - 2x + 3$, then find P(-2).

Solution :

$P(x) = x^2 - 2x + 3$

$P(-2) = (-2)^2 - 2(-2) + 3$

$= 4 + 4 + 3$

$= 11$

2) Find the zeroes of the polynomial $P(x) = x^2 - 3$.

Solution :

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x = \sqrt{3}$ and $-\sqrt{3}$

3) Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$.

Solution :

$x^2 - 2x - 8 = 0$ $\quad$ ($-8x^2$ → $-4x, +2x$)

$x^2 - 4x + 2x - 8 = 0$

$x(x - 4) + 2(x - 4) = 0$

$(x - 4)(x + 2) = 0$

$x - 4 = 0 \quad OR \quad x + 2 = 0$

$x = 4 \quad OR \quad x = -2$

4) Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$ and verify the relationship between the zeroes and the co-efficients.

Solution :

$x^2 + 7x + 10 = 0$ $\quad$ ($10x^2$ → $+5x, +2x$)

$x^2 + 5x + 2x + 10 = 0$

$x(x + 5) + 2(x + 5) = 0$

$(x + 5)(x + 2) = 0$

$x + 5 = 0 \quad OR \quad x + 2 = 0$

$x = -5 \quad OR \quad x = -2$

$Sum\ of\ zeroes = -5 - 2 = -7$

$Product\ of\ zeroes = (-5)(-2) = 10$

$x^2 + 7x + 10 = 0$

$Here\ a = 1, b = 7, c = 10$

$$Sum\ of\ zeroes = \frac{-b}{a} = -\frac{7}{1} = -7$$

$$Product\ of\ zeroes = \frac{c}{a} = \frac{10}{1} = 10$$

Hence verified.

5) If α and β are the zeroes of the polynomial $P(x) = x^2 - 4x + 3$, then find the sum and product of zeroes.

Solution :

$P(x) = x^2 - 4x + 3$

$Here\ a = 1, b = -4, c = 3$

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{1} = 4$$

$$\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$$

6) The sum and product of the zeroes of a quadratic polynomial $P(x) = ax^2 + bx + c$ are – 3 and 2 respectively. Show that $b + c = 5a$.

Solution :

$Sum\ of\ zeroes = -3$

$-\frac{b}{a} = -3$

$\frac{b}{a} = 3$

$b = 3a \ldots\ldots.(1)$

$Product\ of\ zeroes = 2$

$\frac{c}{a} = 2$

$c = 2a \ldots..(2)$

Adding (1) and (2),

$b + c = 3a + 2a$

$= 5a$

7) Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2, respectively.

Solution :

Sum = - 2

Product = 2

The quadratic polynomial is,

$P(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-3)x + 2$

$= x^2 + 3x + 2$

8) If a polynomial $P(x) = x^2 - 4$ is divided by a linear polynomial $(x - 2)$, then find the remainder.

Solution :

$$\frac{x^2 - 4}{x - 2} = \frac{(x-2)(x+2)}{(x-2)} = x + 2$$

$Quotient = x + 2$

$Remainder = 0$

9) Find the quotient and the remainder when $P(x) = 3x^3 + x^2 + 2x + 5$ is divided by $g(x) = x^2 + 2x + 1$.

Solution :

$$\begin{array}{r|l} & 3x - 5 \\ \hline x^2 + 2x + 1 & 3x^3 + x^2 + 2x + 5 \\ & 3x^3 + 6x^2 + 3x \\ & (-)\quad (-)\quad (-) \\ \hline & -5x^2 - x + 5 \\ & -5x^2 - 10x - 5 \\ & (+)\quad (+)\quad (+) \\ \hline & 9x + 10 \end{array}$$

$Quotient = 3x - 5$

$Remainder = 9x + 10$

10) If $P(x) = x^2 + 4x + 4$ and $g(x) = x + 2$ find $q(x)$ and $r(x)$.

Solution :

$$\begin{array}{r|l} & x+2 \\ \hline x+2 & x^2+4x+4 \\ & x^2+2x \\ & (-)\quad(-) \\ \hline & 2x+4 \\ & 2x+4 \\ & (-)\quad(-) \\ \hline & 0 \end{array}$$

$q(x) = x + 2$

$r(x) = 0$

11)If two zeroes of the polynomial $P(x) = x^3 + 2x^2 - 9x - 18$ are 3 and – 3, find other zero of the polynomial.

Solution :

3 and – 3 are zeroes of $P(x)$

$\therefore Factor\ of\ P(x) = (x-3)(x+3)$

$= x^2 - 3^2$

$= x^2 - 9$

$$\begin{array}{r|l} & x+2 \\ \hline x^2+0x-9 & x^3+2x^2-9x-18 \\ & x^3+0x^2-9x \\ & (-)\quad(-)\quad(+) \\ \hline & 2x^2-0x-18 \\ & 2x^2+0x-18 \\ & (-)\quad(-)\quad(+) \\ \hline & 0 \end{array}$$

$(x+2)\ is\ the\ other\ factor\ of\ P(x)$

$\therefore x = -2\ is\ the\ other\ zero\ of\ P(x)$

12)Find all zeroes of the polynomial $P(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

Solution :

$\sqrt{2}$ and $-\sqrt{2}$ are zeroes of $P(x)$.

$\therefore Factor\ of\ P(x) = (x-\sqrt{2})(x+\sqrt{2})$

$= x^2 - (\sqrt{2})^2$

$= x^2 - 2$

$$\begin{array}{r|l} & 2x^2+7x-15 \\ \hline x^2+0x-2 & 2x^4+7x^3-19x^2-14x+30 \\ & 2x^4+0x^3-4x^2 \\ & (-)\quad(-)\quad(+) \\ \hline & 7x^3-15x^2-14x \\ & 7x^3+0x^2-14x \\ & (-)\quad(-)\quad(+) \\ \hline & -15x^2-0x+30 \\ & -15x^2-0x+30 \\ & (+)\quad(+)\quad(-) \\ \hline & 0 \end{array}$$

$2x^2 + 7x - 15\ is\ the\ other\ factor\ of\ P(x).$

$2x^2 + 7x - 15 = 0$

$2x^2 + 10x - 3x - 15 = 0$

$2x(x+5) - 3(x+5) = 0$

$(x+5)(2x-3) = 0$

$x + 5 = 0 \quad OR \quad 2x - 3 = 0$

$x = -5 \quad OR \quad 2x = 3$

$x = \frac{3}{2}$

$\therefore -5\ and\ \frac{3}{2}\ are\ other\ zeroes\ of\ P(x)$

13)On dividing $P(x) = 3x^3 + x^2 + 2x + 5$ by a polynomial $g(x)$, the quotient and remainder obtained are $(3x-5)$ and $(9x+10)$ respectively. Find $g(x)$.

Solution :

By Division Lemma,

$P(x) = g(x) \times q(x) + r(x)$

$$g(x) = \frac{P(x) - r(x)}{q(x)}$$

$$= \frac{3x^3 + x^2 + 2x + 5 - (9x + 10)}{(3x - 5)}$$

$$= \frac{3x^3 + x^2 + 2x + 5 - 9x - 10}{3x - 5}$$

$$= \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$$

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ \hline 3x - 5 & 3x^3 + x^2 - 7x - 5 \\ & 3x^3 - 5x^2 \\ & (-) \quad (+) \\ \hline & \quad 6x^2 - 7x \\ & \quad 6x^2 - 10x \\ & \quad (-) \quad (+) \\ \hline & \qquad 3x - 5 \\ & \qquad 3x - 5 \\ & \qquad (-) \quad (+) \\ \hline & \qquad 0 \end{array}$$

$\therefore g(x) = x^2 + 2x + 1$

14)On dividing $P(x) = x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder obtained are $(x - 2)$ and $(-2x + 4)$ respectively. Find $g(x)$.

Solution :

By Division Lemma,

$P(x) = g(x) \times q(x) + r(x)$

$$g(x) = \frac{P(x) - r(x)}{q(x)}$$

$$= \frac{x^3 - 3x^2 + x + 2 - (-2x + 4)}{(x - 2)}$$

$$= \frac{x^3 - 3x^2 + x + 2 + 2x - 4}{x - 2}$$

$$= \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & (-) \quad (+) \\ \hline & \quad -x^2 + 3x \\ & \quad -x^2 + 2x \\ & \quad (+) \quad (-) \\ \hline & \qquad x - 2 \\ & \qquad x - 2 \\ & \qquad (-) \quad (+) \\ \hline & \qquad 0 \end{array}$$

$\therefore g(x) = x^2 - x + 1$

15)If one zero of the polynomial $P(x) = x^2 - 6x + k$ is twice the other then find the value of k.

Solution :

$P(x) = x^2 - 6x + k.$

$Here\ a = 1, b = -6, c = k$

$Let\ \alpha\ and\ 2\alpha\ are\ two\ zeroes\ of\ P(x).$

$$Sum\ of\ zeroes = -\frac{b}{a}$$

$$\alpha + 2\alpha = \frac{-(-6)}{1}$$

$$3\alpha = 6$$

$$\alpha = \frac{6}{3}$$

$$\alpha = 2$$

$$Product\ of\ zeroes = \frac{c}{a}$$

$$\alpha(2\alpha) = \frac{k}{1}$$

$$2\alpha^2 = k$$

$$2(2^2) = k$$

$$2(4) = k$$
$$8 = k$$
$$\therefore k = 8$$

16)If the sum and product of the zeroes of the polynomial $P(x) = ax^2 - 5x + c$ are equal to 10, then find the values of a and c.

Solution :

$P(x) = ax^2 - 5x + c$

$Sum\ of\ zeroes = 10$

$$-\frac{b}{a} = 10$$
$$\frac{-(-5)}{a} = 10$$
$$5 = 10a$$
$$a = \frac{5}{10}$$
$$a = \frac{1}{2}$$

$Product\ of\ zeroes = 10$

$$\frac{c}{a} = 10$$
$$c = 10a$$
$$= 10 \times \frac{1}{2}$$
$$= 5$$

17)If $(x - 2)$ is a factor of the polynomial $P(x) = x^2 - x + k$ then find the value of k.

Solution :

$P(x) = x^2 - x + k$

$P(2) = 2^2 - 2 + k$

$= 4 - 2 + k$

$= 2 + k$

$(x - 2)\ is\ a\ factor\ of\ P(x)$

$\therefore 2 + k = 0$

$k = -2$

18)If $x = 2$ is one of the zeroes of the polynomial $P(x) = x^2 - 5x + k$ then find the value of k.

Solution :

$P(x) = x^2 - 5x + k$

$P(2) = 2^2 - 5(2) + k$

$= 4 - 10 + k$

$= -6 + k$

$x = 2$ is a zero of $P(x)$

$\therefore P(2) = 0$

$-6 + k = 0$

$k = 6$

19)Find the polynomial of least degree that should be subtracted from the polynomial $P(x) = x^3 - 2x^2 + 3x + 4$ so that it is exactly divisible by $g(x) = x^2 - 3x + 1$.

Solution :

$$\begin{array}{r|l} & x + 1 \\ \hline x^2 - 3x + 1 & x^3 - 2x^2 + 3x + 4 \\ & x^3 - 3x^2 + 1x \\ & (-)\quad (+)\quad (-) \\ \hline & x^2 + 2x + 4 \\ & x^2 - 3x + 1 \\ & (-)\quad (+)\quad (-) \\ \hline & 5x + 3 \end{array}$$

$Polynomial\ to\ be\ subtracted\ is\ 5x + 3$

Lesson-10. Quadratic Equations

An algebraic equation involving a single variable with degree 2 is called a quadratic equation.

Standard form of a quadratic equation :

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where x is the variable and a, b and c are constants.

Solution of a quadratic equation :

The value of the variable x that satisfies the quadratic equation $ax^2 + bx + c = 0$ is called solution or root of the equation.

The quadratic equation $ax^2 + bx + c = 0$ has 2 roots or solutions.

Nature of the roots :

Nature of the roots of the quadratic equation $ax^2 + bx + c = 0$ is determined by the expression $b^2 - 4ac$ and this expression is called discriminant of the quadratic equation.

Discriminant = $b^2 - 4ac$.

If $\mathbf{b^2 - 4ac > 0}$, then the roots are real and distinct.

If $\mathbf{b^2 - 4ac = 0}$, then the roots are real and equal.

If $\mathbf{b^2 - 4ac < 0}$, then the roots are not real and do not exist.

Problems :

1) **Solve the equation $2x^2 - 7x + 3 = 0$ using formula.**

Solution :

$2x^2 - 7x + 3 = 0$

Here a = 2, b = - 7, c = 3.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$= \frac{7 \pm \sqrt{25}}{4}$$

$$= \frac{7 \pm 5}{4}$$

$$x = \frac{7+5}{4} \quad OR \quad x = \frac{7-5}{4}$$

$$x = \frac{12}{4} \quad OR \quad x = \frac{2}{4}$$

$$x = 3 \quad OR \quad x = \frac{1}{2}$$

2) **Find the roots of the quadratic equation $3x^2 - 5x + 2 = 0$ by using formula.**

Solution :

$3x^2 - 5x + 2 = 0$

Here a = 3, b = - 5, c = 2.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{6}$$

$$= \frac{5 \pm \sqrt{1}}{6}$$

$$= \frac{5 \pm 1}{6}$$

$$x = \frac{5+1}{6} \quad OR \quad x = \frac{5-1}{6}$$

$$x = \frac{6}{6} \quad OR \quad x = \frac{4}{6}$$

$$x = 1 \quad OR \quad x = \frac{2}{3}$$

3) Solve $2x^2 + x + 4 = 0$ by sing formula.

Solution :

$2x^2 + x + 4 = 0$

Here a = 2, b = 1, c = 4.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4(2)(4)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{1 - 32}}{4}$$

$$= \frac{-1 \pm \sqrt{-31}}{4}$$

Roots are not real.

4) Find the roots of the equation $6x^2 + 7x - 10 = 0$ using formula.

Solution :

$6x^2 + 7x - 10 = 0$

Here, a = 6, b = 7, c =10 .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{(7)^2 - 4(6)(-10)}}{2(6)}$$

$$= \frac{-7 \pm \sqrt{49 + 240}}{12}$$

$$= \frac{-7 \pm \sqrt{289}}{12}$$

$$= \frac{-7 \pm 17}{12}$$

$$x = \frac{-7 + 17}{12} \quad OR \quad x = \frac{-7 - 17}{12}$$

$$x = \frac{10}{12} \quad OR \quad x = \frac{-24}{12}$$

$$x = \frac{5}{6} \quad OR \quad x = -2$$

5) Find the roots of the quadratic equation $x^2 + 4x + 5 = 0$ by using formula.

Solution :

$x^2 + 4x + 5 = 0$

Here a = 1, b = 4, c = 5.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

Roots are not real.

6) Find the roots of the quadratic equation $2x^2 + x - 4 = 0$ by using formula.

Solution :

$2x^2 + x - 4 = 0$

Here a = 2, b = 1, c = - 4.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{1 + 32}}{4}$$

$$= \frac{-1 \pm \sqrt{33}}{4}$$

$$x = \frac{-1 + \sqrt{33}}{4} \quad OR \quad x = \frac{-1 - \sqrt{33}}{4}$$

7) *Solve* $x + \frac{1}{x} = 3$.

Solution :

$$x + \frac{1}{x} = 3$$

$$\frac{x^2 + 1}{x} = 3$$

$x^2 + 1 = 3x$

$x^2 - 3x + 1 = 0$

Here a = 1, b = -3, c = 1.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

$$x = \frac{3 + \sqrt{5}}{2} \quad OR \quad x = \frac{3 - \sqrt{5}}{2}$$

8) $\boldsymbol{Solve \frac{1}{x} - \frac{1}{x-2} = 3}$

Solution :

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

$$\frac{1(x-2) - 1(x)}{x(x-2)} = 3$$

$$x - 2 - x = 3x(x - 2)$$

$$-2 = 3x^2 - 6x$$

$$3x^2 - 6x + 2 = 0$$

Here a = 3, b = - 6, c = 2.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$= \frac{6 \pm \sqrt{12}}{6}$$

$$= \frac{6 \pm \sqrt{4 \times 3}}{6}$$

$$= \frac{6 \pm 2\sqrt{3}}{6}$$

$$= \frac{2(3 \pm \sqrt{3})}{6}$$

$$x = \frac{3 \pm \sqrt{3}}{3}$$

$$x = \frac{3 + \sqrt{3}}{3} \quad OR \quad x = \frac{3 - \sqrt{3}}{3}$$

9) Find the discriminant of the quadratic equation $\boldsymbol{2x^2 - 4x + 3 = 0}$ and hence find the nature of the roots.

Solution :

$$2x^2 - 4x + 3 = 0$$

Here a = 2, b = - 4 and c = 3.

Discriminant = $b^2 - 4ac$.

$= (-4)^2 - 4(2)(3)$

$= 16 - 24$

$= -8$

< 0

∴ Roots are not real.

10) Find the discriminant of the quadratic equation $\boldsymbol{2x^2 + x + 4 = 0}$ and hence find the nature of the roots.

Solution :

$$2x^2 + x - 4 = 0$$

Here a = 2, b = 1 and c = - 4.

Discriminant = $b^2 - 4ac$.

$= (1)^2 - 4(2)(-4)$

$= 1 + 32$

$= 33$

> 0

∴ Roots are real and distinct.

11) Find the nature of the roots of the quadratic equation $\boldsymbol{2x^2 - 3x + 5 = 0}$.

Solution :

$$2x^2 - 3x + 5 = 0$$

Here a = 2, b = - 3 and c = 5.

Discriminant = $b^2 - 4ac$.

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31$

< 0

∴ Roots are not real.

12)Find the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$.

Solution :

$2x^2 - 6x + 3 = 0$

Here a = 2, b = - 6 and c = 3.

Discriminant = $b^2 - 4ac$.

$= (-6)^2 - 4(2)(3)$

$= 36 - 24$

$= 12$

> 0

∴ Roots are real and distinct.

13)Find the nature of the roots of the quadratic equation $9x^2 - 6x + 1 = 0$.

Solution :

$9x^2 - 6x + 1 = 0$

Here a = 9, b = - 6 and c = 1.

Discriminant = $b^2 - 4ac$.

$= (-6)^2 - 4(9)(1)$

$= 36 - 36$

$= 0$

∴ Roots are real and equal.

14) *Find the nature of the roots of the quadratic equation* $3x^2 - 2x + \frac{1}{3} = 0$

Solution :

$3x^2 - 2x + \frac{1}{3} = 0$

Here a = 3, b = - 2 and c = $\frac{1}{3}$.

Discriminant = $b^2 - 4ac$.

$= (-2)^2 - 4(3)(\frac{1}{3})$

$= 4 - 4$

$= 0$

∴ Roots are real and equal.

15)Find the value of k so that the quadratic equation $2x^2 + kx + 3 = 0$ has equal roots.

Solution :

$2x^2 + kx + 3 = 0$

a = 2, b = k , c = 3

Roots are equal.

∴ $b^2 - 4ac = 0$

$(k)^2 - 4(2)(3) = 0$

$k^2 - 24 = 0$

$k^2 = 24$

$k = \sqrt{24}$

$k = \sqrt{4 \times 6}$

$k = 2\sqrt{6}$

16)Find the value of k for which the quadratic equation $kx(x - 2) + 6 = 0$ has equal roots.

Solution :

$kx(x - 2) + 6 = 0$

$kx^2 - 2kx + 6 = 0$

a = k, b = - 2k, c = 6

Roots are equal.

∴ $b^2 - 4ac = 0$

$(-2k)^2 - 4(k)(6) = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

$4k = 0$ OR $(k - 6) = 0$

$k = 0$ OR $k = 6$

Word Problems :

1) **The length of a rectangular field is 3 times its breadth. If the area of the field is 147 cm², find its length and breadth.**

Solution :

$Let\ breadth\ of\ the\ rectangle\ =\ x$

$\therefore\ Length\ of\ the\ rectangle\ =\ 3x$

$Area\ =\ 147\ cm^2.$

$l\times b\ =\ 147$

$(3x)(x)\ =\ 147$

$3x^2\ =\ 147$

$x^2=\frac{147}{3}$

$x^2\ =\ 49$

$x=\sqrt{49}$

$x\ =\ 7$

∴ Breadth of the rectangle = 7 cm.

Length of the rectangle = 21 cm.

2) **A person distributes Rs. 250 to all children in a class, if the number of children is increased by 25, each child get 50 paise less than the first distribution. Find the number of children.**

Solution :

$Let\ the\ number\ of\ children\ =\ x$

$Then, share\ of\ each\ child=\frac{250}{x}$

$If\ the\ number\ of\ children\ is\ x-25, then$

$Share\ of\ each\ child=\frac{250}{x-25}$

$Difference\ is\ 50\ paise\ or\ Rs.\frac{1}{2}$

$\therefore\frac{250}{x-25}-\frac{250}{x}=\frac{1}{2}$

$\frac{250x-250(x-25)}{(x-25)x}=\frac{1}{2}$

$\frac{250x-250x+6250}{x(x-25)}=\frac{1}{2}$

$\frac{6250}{x^2-25x}=\frac{1}{2}$

$12500=x^2-25x$

$x^2-25x-12500=0$

$x^2-125x+100x-12500=0$

$x(x-125)+100(x-125)=0$

$(x-125)(x+100)=0$

$(x-125)=0\quad OR\quad (x+100)=0$

$x=125\qquad OR\quad x=-100$

$\therefore x=125$

So, the required number of children is 125.

3) **Sanvi purchased some books for Rs. 120. If she purchased 3 more books for the same amount each book would have cost her Rs. 2 less. Find the number of books purchased by Sanvi and the price of the book.**

4) **Some students planned for a picnic. The budget for the food was Rs. 480. As eight of them failed to join the party, the cost of the food for each member would be increased by Rs. 10. Find how many students went for the picnic?**

5) **A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the speed of the train.**

Solution :

$Let\ the\ speed\ of\ the\ train=x$

$Time\ taken=\frac{Distance}{speed}$

$\therefore Time\ taken=\frac{480}{x}$

$If\ the\ speed\ is\ x-8, then$

$Time\ taken = \frac{480}{x-8}$

$Difference\ is\ 3\ hr$

$\therefore \frac{480}{x-8} - \frac{480}{x} = 3$

$\frac{480x - 480(x-8)}{x(x-8)} = 3$

$\frac{480x - 480x + 3840}{x^2 - 8x} = 3$

$\frac{3840}{x^2-8x} = 3$

$3840 = 3(x^2 - 8x)$

$3840 = 3x^2 - 24x$

$3x^2 - 24x - 3840 = 0$

$Divide\ the\ equation\ by\ 3$

$x^2 - 8x - 1280 = 0$

$x^2 - 40x + 32x - 1280 = 0$

$x(x-40) + 32(x-40) = 0$

$(x-40)(x+32) = 0$

$(x-40) = 0 \quad OR \quad (x+32) = 0$

$x = 40 \quad OR \quad x = -32$

$\therefore x = 40$

Speed of the train is 40 km/hr.

6) **A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.**

7) **A motor boat goes down the stream 30 km and again returns to the starting point in a total time of 4 hours and 30 minutes. If the speed of the stream is 5 km/hr, then find the speed of the motor boat in still water.**

Solution :

$Let\ speed\ of\ motor\ boat\ in\ still\ water = x$

$Speed\ of\ the\ stream = 5\ km/hr$

$Speed\ of\ boat\ downstream = x + 5$

$Speed\ of\ boat\ upstream = x - 5$

$Distance\ travelled\ downstream = 30\ km$

$Distance\ travelled\ upstream = 30\ km$

$Time\ taken = \frac{distance}{speed}$

$Time\ taken\ downstream = \frac{30}{x+5}$

$Time\ taken\ upstream = \frac{30}{x-5}$

$Total\ time\ taken = 4hr\ 30\ minutes$

$= 4\frac{1}{2}\ hr$

$= \frac{9}{2}\ hr$

$\therefore \frac{30}{x+5} + \frac{30}{x-5} = \frac{9}{2}$

$\frac{30(x-5) + 30(x+5)}{(x-5)(x+5)} = \frac{9}{2}$

$\frac{30x - 150 + 30x + 150}{x^2 - 5^2} = \frac{9}{2}$

$\frac{60x}{x^2-25} = \frac{9}{2}$

$120x = 9(x^2 - 25)$

$120x = 9x^2 - 225$

$9x^2 - 120x - 225 = 0$

$Divide\ the\ equation\ by\ 3$

$3x^2 - 40x - 75 = 0 \qquad -225x^2$

$3x^2 - 45x + 5x - 75 = 0 \qquad -45x, +5x$

$3x(x-15) + 5(x-15) = 0$

$(x-15)(3x+5) = 0$

$(x-15) = 0 \quad OR \quad (3x+5) = 0$

$x = 15 \quad OR \quad 3x = -5$

$x = -\frac{5}{3}$

$\therefore Speed\ of\ boat\ in\ still\ water\ is\ 15km/h$

8) A motor boat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution :

$Let\ speed\ of\ the\ stream = x$

$Speed\ of\ boat\ in\ still\ water = 18\ km\ /hr$

$Speed\ of\ boat\ downstream = 18 + x$

$Speed\ of\ boat\ upstream = 18 - x$

$Distance\ travelled\ downstream = 24\ km$

$Distance\ travelled\ upstream = 24\ km$

$$Time\ taken = \frac{distance}{speed}$$

$$Time\ taken\ downstream = \frac{24}{18 + x}$$

$$Time\ taken\ upstream = \frac{24}{18 - x}$$

$Difference\ is\ 1\ hr$

$$\therefore \frac{24}{18 - x} - \frac{24}{18 + x} = 1$$

$$\frac{24(18 + x) - 24(18 - x)}{(18 - x)(18 + x)} = 1$$

$$\frac{432 + 24x - 432 + 24x}{18^2 - x^2} = 1$$

$$\frac{48x}{324 - x^2} = 1$$

$48x = 1(324 - x^2)$

$48x = 324 - x^2$

$x^2 + 48x - 324 = 0$

$x^2 + 54x - 6x - 324 = 0$

$x(x + 54) - 6(x + 54) = 0$

$(x + 54)(x - 6) = 0$

$(x + 54) = 0 \quad OR \quad (x - 6) = 0$

$x = -54 \quad OR \quad x = 6$

$\therefore x = 6$

$\therefore Speed\ of\ the\ stream\ is\ 6\ km/hr$.

9) A motor boat whose speed is 15 km/hr in still water goes 30 km downstream and comes back in a total of 4 hours 30 minutes; determine the speed of the stream.

10) A motor boat, whose speed is 9 km/hr in still water, goes 12 km downstream and comes back in a total time of 3 hours. Find the speed of the stream.

11) The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages in years was 124. Find their present ages.

Solution :

$Let\ the\ present\ age\ of\ father = x$

$Then, the\ present\ age\ of\ son = 45 - x$

$5\ years\ ago, the\ age\ of\ father = x - 5$

$the\ age\ of\ son = 40 - x$

$Product\ of\ their\ ages = 124$

$(x - 5)\,(40 - x) = 124$

$40x - x^2 - 200 + 5x - 124 = 0$

$-x^2 + 45x - 324 = 0$

$Multiply\ the\ equation\ by\ (-1)$

$x^2 - 45x + 324 = 0$

$x^2 - 36x - 9x + 324 = 0$

$x(x - 36) - 9(x - 36) = 0$

$(x - 36)(x - 9) = 0$

$(x - 36) = 0 \quad OR \quad (x - 9) = 0$

$x = 36 \quad OR \quad x = 9$

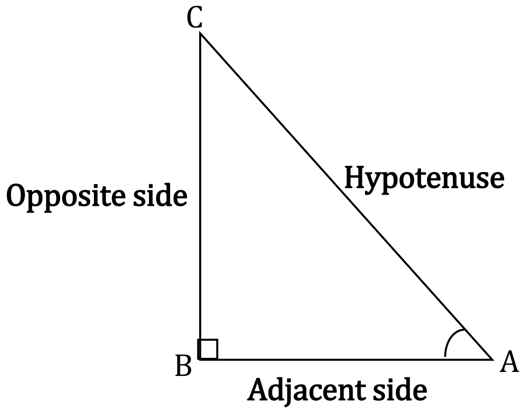
$\therefore The\ present\ age\ of\ father\ is\ 36\ years\ and\ the\ present\ age\ of\ son\ is\ 9\ years.$

Lesson-11. Trigonometry.

Introduction :

Trigonometry is the study of relationships between the sides and angles of a triangle.

Trigonometric ratios :



Consider the right angled triangle Δ ABC in which ∠B $= 90^0$ and ∠A is acute. Then we define the 6 trigonometric ratios as,

$$\text{SinA} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\text{CosA} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\text{TanA} = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

$$\text{CosecA} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

$$\text{SecA} = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$$

$$\text{CotA} = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

Reciprocal Ratios :

$$\text{SinA} = \frac{1}{\text{CosecA}} \quad \text{OR} \quad \text{CosecA} = \frac{1}{\text{SinA}}$$

$$\text{CosA} = \frac{1}{\text{SecA}} \quad \text{OR} \quad \text{SecA} = \frac{1}{\text{CosA}}$$

$$\text{TanA} = \frac{1}{\text{CotA}} \quad \text{OR} \quad \text{CotA} = \frac{1}{\text{TanA}}$$

$$\text{TanA} = \frac{\text{SinA}}{\text{CosA}} \quad \text{OR} \quad \text{CotA} = \frac{\text{CosA}}{\text{SinA}}$$

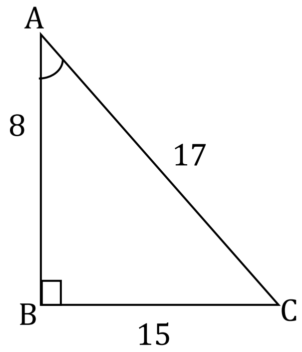
Problems :

1) If 15cotA = 8, then find sinA and secA.

Solution

$15cotA = 8$

$$cotA = \frac{8}{15} = \frac{Adjacent\ side}{Opposite\ side}$$



By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$.

$= 8^2 + 15^2$.

$= 64 + 225$

$= 289$

$AC = \sqrt{289}$

$AC = 17$

$$SinA = \frac{Opposite\ side}{Hypotenuse} = \frac{15}{17}$$

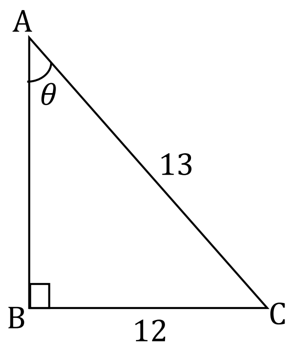
$$SecA = \frac{Hypotenuse}{Adjacent\ side} = \frac{8}{17}$$

3) $\boldsymbol{If\ sin\theta = \frac{12}{13},}$

$\boldsymbol{find\ the\ values\ of\ cos\theta\ and\ tan\theta.}$

Solution :

$$Sin\theta = \frac{12}{13} = \frac{Opposite}{Hypotenuse}$$



By Pythagorus theorem,

$AC^2 = AB^2 + BC^2$.

$13^2 = AB^2 + 12^2$.

$169 = AB^2 + 144$

$AB^2 = 169 - 144$.

$= 25$

$AB = \sqrt{25}$

$AB = 5$

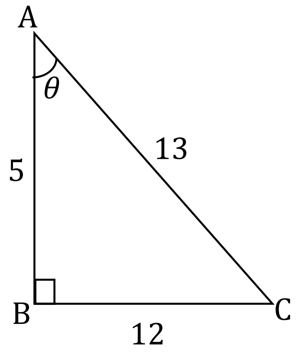
$Cos\theta = \frac{Adjacent\ side}{Hypotenuse} = \frac{5}{13}$

$Tan\theta = \frac{Opposite\ side}{Adjcent\ side} = \frac{12}{5}$

4) If $cos\theta = \frac{5}{13}$, find $\frac{sin\theta + cos\theta}{sin\theta - cos\theta}$.

Solution :

$Cos\theta = \frac{5}{13} = \frac{Adjacent\ side}{Hypotenuse}$



By Pythagorus theorem,

$AC^2 = AB^2 + BC^2$.

$13^2 = 5^2 + BC^2$.

$169 = 25 + BC^2$.

$BC^2 = 169 - 25$.

$= 25$

$BC = \sqrt{144}$

$BC = 12$

$$\frac{sin\theta + cos\theta}{Sin\theta - cos\theta} = \frac{\left(\frac{12}{13} + \frac{5}{13}\right)}{\left(\frac{12}{13} - \frac{5}{13}\right)}$$

$$= \frac{\left(\frac{12+5}{13}\right)}{\left(\frac{12-5}{13}\right)}$$

$$= \frac{17}{7}$$

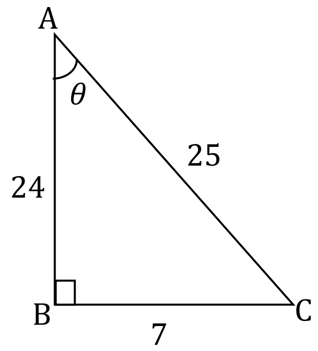
5) If $24\ tan\theta = 7$,

find the values of $sin\theta$ and $cos\theta$.

Solution :

$24\ tan\theta = 7$

$tan\theta = \frac{7}{24} = \frac{Opposite\ side}{Adjacent\ side}$



By Pythagorus theorem,

$AC^2 = AB^2 + BC^2$.

$AC^2 = 24^2 + 7^2$.

$= 576 + 49$

$= 625$.

$AC = \sqrt{625}$

$AC = 25$

$sin\theta = \frac{Opposite\ side}{Hypotenuse} = \frac{7}{25}$

$Cos\theta = \frac{Adjacent\ side}{Hypotenuse} = \frac{24}{25}$

Trigonometric Ratios of Standard angles :

θ	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND

Problems :

1) $\boldsymbol{If\ \sqrt{3}tan\theta = 1\ and\ \theta\ is\ acute,}$ $\boldsymbol{find\ the\ value\ of\ sin3\theta + cos2\theta.}$

Solution :

$\sqrt{3}tan\theta = 1$

$tan\theta = \frac{1}{\sqrt{3}}$

$tan\theta = tan30^0$

$\therefore\ \theta = 30^0$

$sin3\theta + cos2\theta = sin3(30^0) + cos2(30^0)$

$= sin90^0 + cos60^0$

$= 1 + \frac{1}{2}$

$= \frac{3}{2}$

2) $\boldsymbol{Find\ the\ value\ of}$ $\boldsymbol{sin30^0 . cos60^0 - tan^2 45^0.}$

Solution :

$sin30^0 . cos60^0 - tan^2 45^0$

$= \frac{1}{2} \times \frac{1}{2} - (1)^2$

$= \frac{1}{4} - 1$

$= -\frac{3}{4}$

8) $\boldsymbol{Evaluate}$:

$\boldsymbol{2\ tan^2 45^0 + cos^2 30^0 - sin^2 30^2}$

Solution :

$2\ tan^2 45^0 + cos^2 30^0 - sin^2 60^0$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2(1) + \frac{3}{4} - \frac{3}{4}$

$= 2$

9) $\boldsymbol{If\ A = 60^0, B = 30^0,}$ $\boldsymbol{then\ verify\ that}$ $\boldsymbol{cos(A + B) = cosA.cosB - sinA.sinB.}$

Solution :

$LHS = \cos(A + B)$

$= \cos(60^0 + 30^0)$

$= cos90^0$

$= 0$

$RHS = cosA\ cosB - sinA\ sinB$

$= cos60^0\ cos30^0 - sin60^0\ sin30^0$

$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$

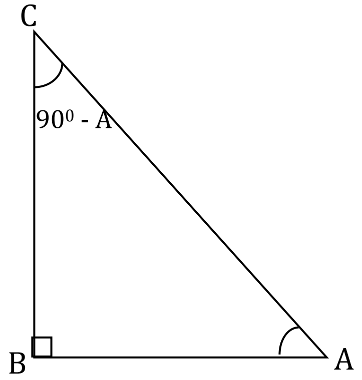
$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$= 0$

$\therefore LHS = RHS$

Hence Proved.

Trigonometric ratios of Complementary angles .



$Sin(90^0 - A) = CosA$

$Cos(90^0 - A) = sinA$

$tan(90^0 - A) = cotA$

$cosec(90^0 - A) = secA$

$sec(90^0 - A) = cosecA$

$Ccot(90^0 - A) = tanA$

Problems :

1) If tan2A =cot(A-18⁰), where 2A is an acute angle, find the value of A.

Solution :

$tan2A = \cot(A - 18^0)$

$\cot(90^0 - 2A) = \cot(A - 18^0)$

$90^0 - 2A = A - 18^0$

$2A + A = 90^0 + 18^0$

$3A = 108^0$

$A = \frac{108^0}{3}$

$A = 36^0$

2) ***Prove that***

$$\frac{\boldsymbol{cos\ 70^0}}{\mathbf{sin\ 20^0}} + \frac{\mathbf{cos\ 55^0}.\boldsymbol{cosec\ 35^0}}{\mathbf{tan\ 5^0.tan\ 25^0}.\boldsymbol{tan65^0}.\boldsymbol{tan85^0}} = \mathbf{2}$$

Solution :

$$LHS = \frac{cos\ 70^0}{\sin 20^0} + \frac{\cos 55^0 . cosec\ 35^0}{\tan 5^0 . \tan 25^0 . tan65^0 . tan85^0}$$

$$= \frac{sin20^0}{\sin 20^0} + \frac{\sin 35^0 . cosec35^0}{\tan5^0 . \tan25^0 . \cot25^0 . \cot5^0}$$

$$= \frac{sin20^0}{sin20^0} + \frac{sin35^0 \times \frac{1}{sin35^0}}{\tan 5^0 . \tan 25^0 . \frac{1}{tan25} \cdot \frac{1}{tan5^0}}$$

$$= 1 + \frac{1}{1}$$

$$= 1 + 1$$

$$= 2$$

$$= RHS$$

Hence Proved.

Trigonometric Identities :

1) $sin^2\theta + cos^2\theta = 1$

$OR\ \ sin^2\theta = 1 - cos^2\theta$

$OR\ \ cos^2\theta = 1 - sin^2\theta$

2) $1 + tan^2\theta = sec^2\theta$

$OR\ \ tan^2\theta = sec^2\theta - 1$

$OR\ \ sec^2\theta - tan^2\theta = 1$

3) $1 + cot^2\theta = cosec^2\theta$

$OR\ \ cot^2\theta = cosec^2\theta - 1$

$OR\ \ cosec^2\theta - cot^2\theta = 1$

Problems :

1) ***Prove that***

$$\frac{\mathbf{sin(90 - \boldsymbol{A})}}{\mathbf{1 - \boldsymbol{tanA}}} + \frac{\mathbf{cos(90 - \boldsymbol{A})}}{\mathbf{1 - \boldsymbol{cotA}}} = \boldsymbol{cosA + sinA}.$$

Solution :

$$LHS = \frac{\sin(90 - A)}{1 - tanA} + \frac{\cos(90 - A)}{1 - cotA}$$

$$= \frac{cosA}{1 - tanA} + \frac{sinA}{1 - cotA}$$

$$= \frac{cosA}{\left(1 - \frac{sinA}{cosA}\right)} + \frac{sinA}{\left(1 - \frac{cosA}{sinA}\right)}$$

$$= \frac{cosA}{\left(\frac{cosA - sinA}{cosA}\right)} + \frac{sinA}{\left(\frac{sinA - cosA}{sinA}\right)}$$

$$= \frac{cos^2A}{cosA - sinA} + \frac{sin^2A}{sinA - cosA}$$

$$= \frac{cos^2A}{cosA - sinA} - \frac{sin^2A}{cosA - sinA}$$

$$= \frac{cos^2A - sin^2A}{cosA - sinA}$$

$$= \frac{(cosA + sinA)(cosA - sinA)}{(cosA - sinA)}$$

$= cosA + sinA$

$= RHS$

Hence Proved.

2) *Prove that*

$$\frac{1+cos\theta}{sin\theta}+\frac{sin\theta}{1+cos\theta}=2cosec\theta.$$

Solution :

$$LHS=\frac{1+cos\theta}{sin\theta}+\frac{sin\theta}{1+cos\theta}$$

$$=\frac{(1+cos\theta)^2+sin^2\theta}{sin\theta(1+cos\theta)}$$

$$=\frac{1+2(1)(cos\theta)+cos^2\theta+sin^2\theta}{sin\theta(1+cos\theta)}$$

$$=\frac{1+2cos\theta+cos^2\theta+(1-cos^2\theta)}{sin\theta(1+cos\theta)}$$

$$=\frac{2+2cos\theta}{sin\theta(1+cos\theta)}$$

$$=\frac{2(1+cos\theta)}{sin\theta(1+cos\theta)}$$

$$=\frac{2}{sin\theta}$$

$$=2cosec\theta$$

3) *Prove that*

$$\frac{tan\theta}{1-cot\theta}+\frac{cot\theta}{1-tan\theta}=1+sec\theta.cosec\theta.$$

Solution :

$$\frac{tan\theta}{1-cot\theta}+\frac{cot\theta}{1-tan\theta}$$

$$=\frac{\frac{sin\theta}{cos\theta}}{1-\frac{cos\theta}{sin\theta}}+\frac{\frac{cos\theta}{sin\theta}}{1-\frac{sin\theta}{cos\theta}}$$

$$=\frac{\frac{sin\theta}{cos\theta}}{\frac{sin\theta-cos\theta}{sin\theta}}+\frac{\frac{cos\theta}{sin\theta}}{\frac{cos\theta-sin\theta}{cos\theta}}$$

$$=\frac{sin^2\theta}{cos\theta(sin\theta-cos\theta)}+\frac{cos^2\theta}{sin\theta(cos\theta-sin\theta)}$$

$$=\frac{sin^2\theta}{cos\theta(sin\theta-cos\theta)}-\frac{cos^2\theta}{sin\theta(sin\theta-cos\theta)}$$

$$=\frac{1}{(sin\theta-cos\theta)}\left[\frac{sin^2\theta}{cos\theta}-\frac{cos^2\theta}{sin\theta}\right]$$

$$=\frac{1}{(sin\theta-cos\theta)}\left[\frac{sin^3\theta-cos^3\theta}{sin\theta cos\theta}\right]$$

$$=\frac{(sin\theta-cos\theta)(sin^2\theta+sin\theta cos\theta+cos^2\theta)}{(sin\theta-cos\theta)(sin\theta cos\theta)}$$

$$=\frac{(sin^2\theta+sin\theta cos\theta+cos^2\theta)}{sin\theta cos\theta}$$

$$=\frac{1+sin\theta cos\theta}{sin\theta cos\theta}$$

$$=\frac{1}{sin\theta cos\theta}+\frac{sin\theta cos\theta}{sin\theta cos\theta}$$

$$=\frac{1}{sin\theta}\frac{1}{cos\theta}+1$$

$$=cosec\theta\ sec\theta+1$$

Hence Proved.

4) *Prove that,*

$$\frac{cotA-cosA}{cotA+cosA}=\frac{cosecA-1}{cosecA+1}.$$

Solution :

$$LHS=\frac{cotA-cosA}{cotA+cosA}$$

$$=\frac{\left(\frac{cosA}{sinA}-cosA\right)}{\left(\frac{cosA}{sinA}+cosA\right)}$$

$$=\frac{cosA\left(\frac{1}{sinA}-1\right)}{cosA\left(\frac{1}{sinA}+1\right)}$$

$$=\frac{cosecA-1}{cosecA+1}$$

Hence Proved.

5) *Prove that*

$$(sinA+cosecA)^2+(cosA+secA)^2 = 7+tan^2A+cot^2A.$$

Solution :

$(sinA+cosecA)^2+(cosA+secA)^2$

$= sin^2A+cosec^2A+2sinA.cosecA+$

$cos^2A+sec^2A+2cosA.secA$

$= sin^2A + cosec^2A + 2 + cos^2A + sec^2A + 2$

$= 1 + 4 + cosec^2A + sec^2A$

$= 5 + 1 + cot^2A + 1 + tan^2A$

$= 7 + tan^2A + cot^2A$

Hence Proved.

6) *Prove that,*

$$\mathbf{\frac{1 + sin\theta}{1 - sin\theta} - \frac{1 - sin\theta}{1 + sin\theta} = 4\ tan\theta.sec\theta.}$$

Solution :

$$\frac{1 + sin\theta}{1 - sin\theta} - \frac{1 - sin\theta}{1 + sin\theta}$$

$$= \frac{(1 + sin\theta)^2 - (1 - sin\theta)^2}{(1 - sin\theta)(1 + sin\theta)}$$

$$= \frac{1 + 2sin\theta + sin^2\theta - (1 - 2sin\theta + sin^2\theta}{1^2 - sin^2\theta}$$

$$= \frac{1 + 2sin\theta + sin^2\theta - 1 + 2sin\theta - sin^2\theta}{cos^2\theta}$$

$$= \frac{4sin\theta}{cos^2\theta}$$

$$= 4.\frac{1}{cos\theta}.\frac{sin\theta}{cos\theta}$$

$$= 4sec\theta.tan\theta$$

Hence Proved.

7) *Prove that,*

$$\mathbf{\frac{\sin(90 - \theta)}{1 + sin\theta} + \frac{cos\theta}{1 - \cos(90 - \theta)} = 2sec\theta.}$$

Solutiuon

$$\frac{\sin(90 - \theta)}{1 + sin\theta} + \frac{cos\theta}{1 - \cos(90 - \theta)}$$

$$= \frac{cos\theta}{1 + sin\theta} + \frac{cos\theta}{1 - sin\theta}$$

$$= \frac{cos\theta(1 - sin\theta) + cos\theta(1 + sin\theta)}{(1 + sin\theta)(1 - sin\theta)}$$

$$= \frac{cos\theta - sin\theta\ cos\theta + cos\theta + sin\theta\ cos\theta}{1^2 - sin^2\theta}$$

$$= \frac{2cos\theta}{1 - sin^2\theta}$$

$$= \frac{2cos\theta}{cos^2\theta}$$

$$= \frac{2}{cos\theta}$$

$$= 2\frac{1}{cos\theta}$$

$$= 2sec\theta$$

Hence Proved.

8) *Prove that,*

$$\mathbf{\frac{1 + cos\theta}{1 - cos\theta} = (cosec\theta + cot\theta)^2.}$$

Solution :

$$LHS = \frac{1 + cos\theta}{1 - cos\theta}$$

$$= \frac{1 + cos\theta}{1 - cos\theta} \times \frac{1 + cos\theta}{1 + cos\theta}$$

$$= \frac{(1 + cos\theta)^2}{1^2 - cos^2\theta}$$

$$= \frac{1^2 + 2(1)(cos\theta) + cos^2\theta}{1 - cos^2\theta}$$

$$= \frac{1 + 2cos\theta + cos^2\theta}{sin^2\theta}$$

$$= \frac{1}{sin^2\theta} + \frac{2cos\theta}{sin^2\theta} + \frac{cos^2\theta}{sin^2\theta}$$

$$= cosec^2\theta + 2.\frac{cos\theta}{sin\theta}.\frac{1}{sin\theta} + cot^2\theta$$

$$= cosec^2\theta + 2cot\theta.cosec\theta + cot^2\theta$$

$$= (cosec\theta + cot\theta)^2$$

Hence Proved.

9) *Prove that,*

$$\mathbf{\frac{sin\theta}{1 - cos\theta} + \frac{tan\theta}{1 + cos\theta} = cot\theta + sec\theta.cosec\theta.}$$

Solution :

$$\frac{sin\theta}{1 - cos\theta} + \frac{tan\theta}{1 + cos\theta}$$

$$= \frac{sin\theta(1 + cos\theta) + tan\theta(1 - cos\theta)}{(1 - cos\theta)(1 + cos\theta)}$$

$$= \frac{sin\theta + sin\theta\ cos\theta + tan\theta - tan\theta\ cos\theta}{1^2 - cos^2\theta}$$

$$= \frac{sin\theta + sin\theta\ cos\theta + tan\theta - \frac{sin\theta}{cos\theta}\ cos\theta}{1 - cos^2\theta}$$

$$= \frac{sin\theta + sin\theta\, cos\theta + tan\theta - sin\theta}{sin^2\theta}$$

$$= \frac{sin\theta\, cos\theta + tan\theta}{sin^2\theta}$$

$$= \frac{sin\theta\, cos\theta}{sin^2\theta} + \frac{tan\theta}{sin^2\theta}$$

$$= \frac{cos\theta}{sin\theta} + \frac{sin\theta}{cos\theta} \times \frac{1}{sin^2\theta}$$

$$= cot\theta + \frac{1}{cos\theta} \times \frac{1}{sin\theta}$$

$$= cot\theta + sec\theta\, cosec\theta$$

Hence Proved.

10) *Prove that,*

$$(\mathbf{1 + cotA - cosecA})\ (\mathbf{1 + tanA + secA}) = \mathbf{2}$$

Solution :

$$(1 + cotA - cosecA)(1 + tanA + secA)$$

$$= \left(1 + \frac{cosA}{sinA} - \frac{1}{sinA}\right)\left(1 + \frac{sinA}{cosA} + \frac{1}{cosA}\right)$$

$$= \left(1 + \frac{cosA - 1}{sinA}\right)\left(1 + \frac{sinA + 1}{cosA}\right)$$

$$= \left(\frac{sinA + cosA - 1}{sinA}\right)\left(\frac{cosA + sinA + 1}{cosA}\right)$$

$$= \frac{(sinA + cosA)^2 - 1^2}{sinA\, cosA}$$

$$= \frac{sin^2A + cos^2A + 2sinA\, cosA - 1}{sinA\, cosA}$$

$$= \frac{1 + 2sinA\, cosA - 1}{sinA\, cosA}$$

$$= \frac{2sinA\, cosA}{sinA\, cosA}$$

$$= 2$$

11) *Prove that*

$$\mathbf{cot\theta . cos\theta + sin\theta = cosec\theta.}$$

Solution :

$$LHS = cot\theta . cos\theta + sin\theta$$

$$= \frac{cos\theta}{sin\theta} . cos\theta + sin\theta$$

$$= \frac{cos^2\theta}{sin\theta} + sin\theta$$

$$= \frac{cos^2\theta + sin^2\theta}{sin\theta}$$

$$= \frac{1}{sin\theta}$$

Hence Proved.

12) *Prove that*

$$\mathbf{tan^2A - sin^2A = tan^2A . sin^2A.}$$

Solution :

$$LHS = tan^2A - sin^2A$$

$$= \frac{sin^2A}{cos^2A} - sin^2A$$

$$= \frac{sin^2A - sin^2A\, cos^2A}{cos^2A}$$

$$= \frac{sin^2A(1 - cos^2A)}{cos^2A}$$

$$= \frac{sin^2A\, sin^2A}{cos^2A}$$

$$= \frac{sin^2A}{cos^2A} . sin^2A$$

$$= tan^2A\, sin^2A$$

Hence Proved.

13) *Prove that*

$$\mathbf{\frac{1 - tan^2A}{1 + tan^2A} = cos^2A - sin^2A}$$

Solution :

$$LHS = \frac{1 - tan^2A}{1 + tan^2A}$$

$$= \frac{\left(1 - \frac{sin^2A}{cos^2A}\right)}{\left(1 + \frac{sin^2A}{cos^2A}\right)}$$

$$= \frac{\left(\frac{cos^2A - sin^2A}{cos^2A}\right)}{\left(\frac{cos^2A + sin^2A}{cos^2A}\right)}$$

$$= \frac{cos^2A - sin^2A}{cos^2A + sin^2A}$$

$$= \frac{cos^2A - sin^2A}{1}$$

$$= cos^2A - sin^2A$$

14) *Prove that* $\mathbf{\frac{1-tan^2A}{1+tan^2A}=2cos^2A-1}$

Solution :

$$LHS=\frac{1-tan^2A}{1+tan^2A}$$

$$=\frac{\left(1-\frac{sin^2A}{cos^2A}\right)}{\left(1+\frac{sin^2A}{cos^2A}\right)}$$

$$=\frac{\left(\frac{cos^2A-sin^2A}{cos^2A}\right)}{\left(\frac{cos^2A+sin^2A}{cos^2A}\right)}$$

$$=\frac{cos^2A-sin^2A}{cos^2A+sin^2A}$$

$$=\frac{cos^2A-sin^2A}{1}$$

$$=cos^2A-sin^2A$$

$$=cos^2A-(1-cos^2A)$$

$$=cos^2A-1+cos^2A$$

$$=2cos^2A-1$$

15) *Prove that* $\mathbf{\frac{1-tan^2A}{1+tan^2A}=1-2sin^2A}$

$$LHS=\frac{1-tan^2A}{1+tan^2A}$$

$$=\frac{\left(1-\frac{sin^2A}{cos^2A}\right)}{\left(1+\frac{sin^2A}{cos^2A}\right)}$$

$$=\frac{\left(\frac{cos^2A-sin^2A}{cos^2A}\right)}{\left(\frac{cos^2A+sin^2A}{cos^2A}\right)}$$

$$=\frac{cos^2A-sin^2A}{cos^2A+sin^2A}$$

$$=\frac{cos^2A-sin^2A}{1}$$

$$=cos^2A-sin^2A$$

$$=1-sin^2A-sin^2A$$

$$=1-2sin^2A$$

Hence Proved.

16) *Prove that*,

$$\mathbf{\frac{\sin(90-\theta)}{cosec(90-\theta)-\cot(90-\theta)}=1+\sin\theta}$$

Solution :

$$\text{LHS}=\frac{\sin(90-\theta)}{\text{cosec}(90-\theta)-\cot(90-\theta)}$$

$$=\frac{cos\theta}{sec\theta-tan\theta}$$

$$=\frac{cos\theta}{\frac{1}{cos\theta}-\frac{sin\theta}{cos\theta}}$$

$$=\frac{cos\theta}{\frac{1-sin\theta}{cos\theta}}$$

$$=\frac{cos^2\theta}{1-sin\theta}$$

$$=\frac{1-sin^2\theta}{1-sin\theta}$$

$$=\frac{(1+sin\theta)(1-sin\theta)}{(1-sin\theta)}$$

$$=1+sin\theta$$

Hence Proved.

17) *Prove that*,

$$\mathbf{\sqrt{\frac{1+cos\theta}{1-cos\theta}}=cosec\theta+cot\theta}$$

Solution :

$$LHS=\sqrt{\frac{1+cos\theta}{1-cos\theta}}$$

$$=\sqrt{\frac{1+cos\theta}{1-cos\theta}}$$

$$=\sqrt{\frac{1+cos\theta}{1-cos\theta}\times\frac{1+cos\theta}{1+cos\theta}}$$

$$=\sqrt{\frac{(1+cos\theta)^2}{1^2-cos^2\theta}}$$

$$= \sqrt{\frac{(1+cos\theta)^2}{sin^2\theta}}$$

$$= \frac{1+cos\theta}{sin\theta}$$

$$= \frac{1}{sin\theta} + \frac{cos\theta}{sin\theta}$$

$$= cosec\theta + cot\theta$$

Hence Proved.

18) *Prove that*

$$(\mathbf{1} - \boldsymbol{sin^2A})(\mathbf{1} + \boldsymbol{tan^2A}) = \mathbf{1}.$$

Solution :

$$LHS = (1 - sin^2A)(1 + tan^2A)$$

$$= (1 - sin^2A)\left(1 + \frac{sin^2A}{cos^2A}\right)$$

$$= (cos^2A)\left(\frac{cos^2A + sin^2A}{cos^2A}\right)$$

$$= (cos^2A)\left(\frac{1}{cos^2A}\right)$$

$$= 1$$

Hence Proved.

19) *Prove that,*

$$\frac{\boldsymbol{sin\theta}}{\mathbf{1}+\boldsymbol{cos\theta}} + \frac{\mathbf{1}+\boldsymbol{cos\theta}}{\boldsymbol{sin\theta}} = \mathbf{2}\boldsymbol{cosec\theta}$$

Solution :

$$LHS = \frac{sin\theta}{1+cos\theta} + \frac{1+cos\theta}{sin\theta}$$

$$= \frac{sin^2\theta + (1+cos\theta)^2}{(1+cos\theta)sin\theta}$$

$$= \frac{sin^2\theta + 1^2 + cos^2\theta + 2(1)(cos\theta)}{(1+cos\theta)sin\theta}$$

$$= \frac{sin^2\theta + 1 + cos^2\theta + 2cos\theta}{(1+cos\theta)sin\theta}$$

$$= \frac{1+1+2cos\theta}{(1+cos\theta)sin\theta}$$

$$= \frac{2+2cos\theta}{(1+cos\theta)sin\theta}$$

$$= \frac{2(1+cos\theta)}{(1+cos\theta)sin\theta}$$

$$= \frac{2}{sin\theta}$$

$$= 2cosec\theta.$$

Hence Proved.

20) *Prove that,*

$$\frac{\boldsymbol{sec\theta} - \boldsymbol{tan\theta}}{\boldsymbol{sec\theta} + \boldsymbol{tan\theta}} = \mathbf{1} - \mathbf{2}\boldsymbol{sec\theta}.\boldsymbol{tan\theta} + \mathbf{2}\boldsymbol{tan^2\theta}$$

Solution :

$$LHS = \frac{sec\theta - tan\theta}{sec\theta + tan\theta}$$

$$= \frac{\left(\frac{1}{cos\theta} - \frac{sin\theta}{cos\theta}\right)}{\left(\frac{1}{cos\theta} + \frac{sin\theta}{cos\theta}\right)}$$

$$= \frac{\left(\frac{1-sin\theta}{cos\theta}\right)}{\left(\frac{1+sin\theta}{cos\theta}\right)}$$

$$= \frac{1-sin\theta}{1+sin\theta}$$

$$= \frac{1-sin\theta}{1+sin\theta} \times \frac{1-sin\theta}{1-sin\theta}$$

$$= \frac{(1-sin\theta)^2}{1^2 - sin^2\theta}$$

$$= \frac{1^2 + sin^2\theta - 2(1)(sin\theta)}{1 - sin^2\theta}$$

$$= \frac{1 + sin^2\theta - 2sin\theta}{cos^2\theta}$$

$$= \frac{1}{cos^2\theta} + \frac{sin^2\theta}{cos^2\theta} - \frac{2sin\theta}{cos^2\theta}$$

$$= sec^2\theta + tan^2\theta - 2\frac{sin\theta}{cos\theta}.\frac{1}{cos\theta}$$

$$= sec^2\theta + tan^2\theta - 2tan\theta\ sec\theta$$

$$= (sec\theta - tan\theta)^2$$

Hence Proved.

21) *If* $\boldsymbol{x = p\ tan\theta + q\ sec\theta}$ ***and***

$\boldsymbol{y = p\ sec\theta + q\ tan\theta}$, ***then***

Prove that, $\boldsymbol{x^2 - y^2 = q^2 - p^2}$.

Solution :

$x^2 - y^2$

$= (p\ tan\theta + q\ sec\theta)^2 - (p\ sec\theta + q\ tan\theta)^2$

$= p^2tan^2\theta + q^2sec^2\theta + 2pq.tan\theta.sec\theta$

$\quad - p^2sec^2\theta - q^2tan^2\theta - 2pq.tan\theta.sec\theta$

$= p^2(tan^2\theta - sec^2\theta) + q^2(sec^2\theta - tan^2\theta)$

$= p^2(-1) + q^2(1)$

$= -p^2 + q^2$

$= q^2 - p^2$

$= RHS$

19) *Prove that,*

$$\frac{cot^2(90-\theta)}{tan^2\theta - 1} + \frac{cosec^2\theta}{sec^2\theta - cosec^2\theta} = \frac{1}{sin^2\theta - cos^2\theta}$$

Solution :

$$LHS = \frac{cot^2(90-\theta)}{tan^2\theta - 1} + \frac{cosec^2\theta}{sec^2\theta - cosec^2\theta}$$

$$= \frac{tan^2\theta}{tan^2\theta - 1} + \frac{cosec^2\theta}{sec^2\theta - cosec^2\theta}$$

$$= \frac{\left(\frac{sin^2\theta}{cos^2\theta}\right)}{\left(\frac{sin^2\theta}{cos^2\theta} - 1\right)} + \frac{\left(\frac{1}{sin^2\theta}\right)}{\left(\frac{1}{cos^2\theta} - \frac{1}{sin^2\theta}\right)}$$

$$= \frac{\left(\frac{sin^2\theta}{cos^2\theta}\right)}{\left(\frac{sin^2\theta - cos^2\theta}{cos^2\theta}\right)} + \frac{\left(\frac{1}{sin^2\theta}\right)}{\left(\frac{sin^2\theta - cos^2\theta}{cos^2\theta.sin^2\theta}\right)}$$

$$= \frac{sin^2\theta}{sin^2\theta - cos^2\theta} + \frac{cos^2\theta}{sin^2\theta - cos^2\theta}$$

$$= \frac{sin^2\theta + cos^2\theta}{sin^2\theta - cos^2\theta}$$

$$= \frac{1}{sin^2\theta - cos^2\theta}$$

Lesson - 12.

Applications of Trigonometry.

Heights and Distances :

One of the applications of Trigonometry is to find the heights and distances of various objects, without actually measuring them.

Line of sight :

Line of sight is the line drawn from the eye of an observer to the point in the object.

Angle of elevation :

Angle of elevation is the angle formed by the line of sight with the horizontal when the object is viewed above the horizontal.

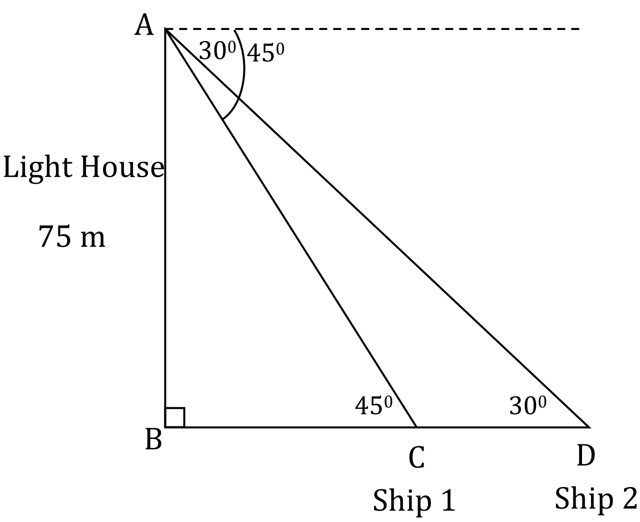
Angle of depression :

Angle of depression is the angle formed by the line of sight with the horizontal when the object is viewed below the horizontal.

Problems :

1) **As observed from top of a 75 m high light house, angles of depressions of two ships are 30^0 and 45^0. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships.**

Solution :



$In\ \Delta\ ABC$, $\tan 45^0 = \frac{AB}{BC}$

$$1 = \frac{75}{BC}$$

$$BC = 75\ m$$

$In\ \Delta\ ABD$, $\tan 30^0 = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{75}{BD}$$

$$BD = 75\sqrt{3}\ m$$

$CD = BD - BC$

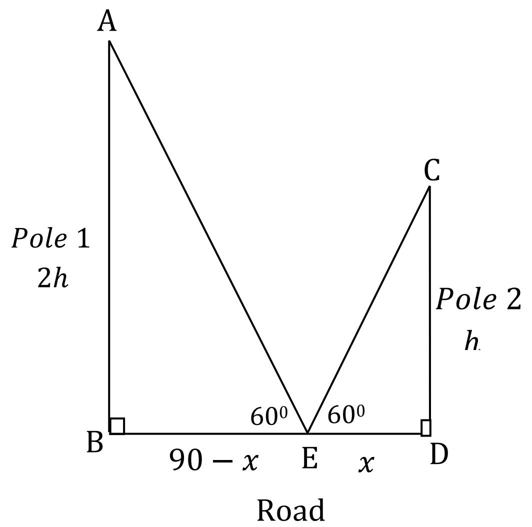
$= 75\sqrt{3} - 75$

$= 75(\sqrt{3} - 1)\ m$

$\therefore$ *Distance between the ships is* $75(\sqrt{3} - 1)\ m.$

2) A man observes two vertical poles which are fixed opposite to each other on either side of the road. If the width of the road is 90 feet and heights of the poles are in the ratio 1 : 2, also the angle of elevation of their tops from a point between the line joining the foot of the poles on the road is 60^0. Find the heights of the poles.

Solution :



$In\ \Delta\ CDE$, $\tan 60^0 = \frac{DC}{DE}$

$$\sqrt{3} = \frac{h}{x}$$

$$x\sqrt{3} = h$$

$$x = \frac{h}{\sqrt{3}}$$

$In\ \Delta\ ABE$, $\tan 60^0 = \frac{AB}{BE}$

$$\sqrt{3} = \frac{2h}{90 - x}$$

$$2h = (90 - x)\sqrt{3}$$

$$2h = 90\sqrt{3} - x\sqrt{3}$$

$$2h = 90\sqrt{3} - \left(\frac{h}{\sqrt{3}}\right)\sqrt{3}$$

$$2h = 90\sqrt{3} - h$$

$$3h = 90\sqrt{3}$$

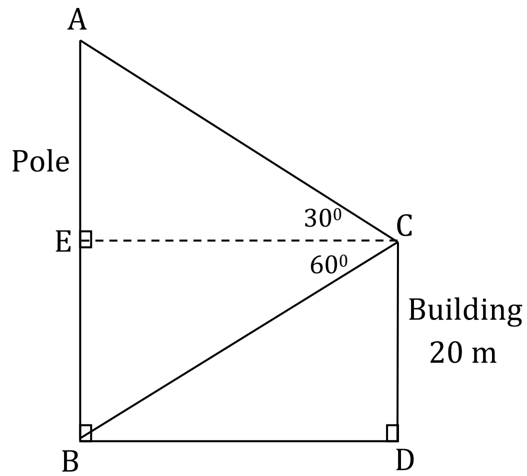
$$h = \frac{90\sqrt{3}}{3}$$

$$h = 30\sqrt{3}\ m$$

$\therefore$ *Heights of poles are* $30\sqrt{3}\ m$ *and* $60\sqrt{3}\ m.$

3) From the top of a building 20 meter high, the angle of elevation of the top of a vertical pole is 30^0, and the angle of depression of the foot of the same pole is 60^0. Find the height of the pole.

Solution :



$$In\ \Delta\ AEC, \quad \tan 30^0 = \frac{AE}{CE}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{CE}$$

$$CE = AE\sqrt{3}$$

$$In\ \Delta\ BCE, \quad \tan 60^0 = \frac{BE}{CE}$$

$$\sqrt{3} = \frac{20}{AE\sqrt{3}}$$

$$\sqrt{3} \times AE\sqrt{3} = 20$$

$$AE(\sqrt{3})^2 = 20$$

$$AE(3) = 20$$

$$AE = \frac{20}{3}$$

$$= 6\frac{2}{3}\ m$$

$$\therefore Height\ of\ the\ pole = AE + BE$$

$$= \frac{20}{3} + 20$$

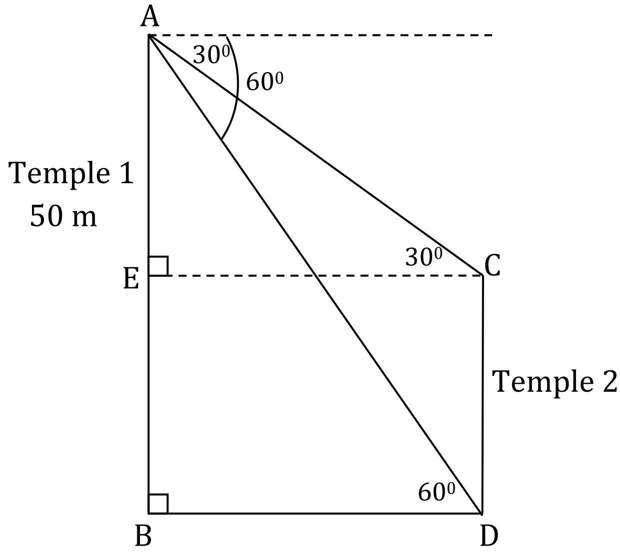
$$= \frac{20 + 60}{3}$$

$$= \frac{80}{3}$$

$$= 26\frac{2}{3}\ m$$

4) There are two temples one on each bank of a river just opposite to each other. One temple is 50 m high. From the top of this temple, the angles of depression of the top and foot of other temple are 30^0 and 60^0 respectively. Find the width of the river and the height of the other temple.

Solution :



$$In\ \Delta\ ABD, \quad \tan 60^0 = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{50}{BD}$$

$$\sqrt{3} \times BD = 50$$

$$BD = \frac{50}{\sqrt{3}}$$

$$In\ \Delta\ ACE, \quad \tan 30^0 = \frac{AE}{EC}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{BD}$$

$$BD = AE\sqrt{3}$$

$$AE = \frac{BD}{\sqrt{3}}$$

$$AE = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$AE = \frac{50}{3}$$

$$AE = 16\frac{2}{3}\ m$$

$$\therefore Width\ of\ the\ river = BD$$

$$= \frac{50}{\sqrt{3}}$$

$$= \frac{50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{50\sqrt{3}}{3}\ m$$

$Height\ of\ the\ other\ temple = CD$

$= AB - AE$

$= 50 - \frac{50}{3}$

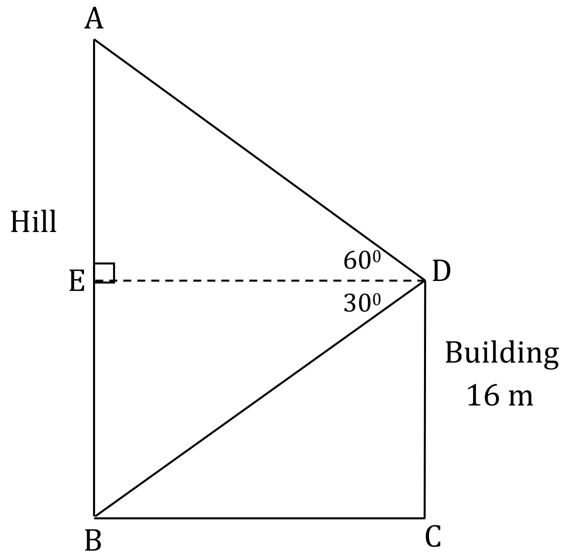
$= \frac{150 - 50}{3}$

$= \frac{100}{3}$

$= 33\frac{1}{3}\ m$

5) From the top of a building 16 m high, the angular elevation of the top of a hill is 60⁰ and the angular depression of the foot of the hill is 30⁰. Find the height of the hill.

Solution :



$In\ \Delta\ BDE,\quad \tan 30^0 = \frac{BE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{16}{DE}$

$DE = 16\sqrt{3}\ m$

$In\ \Delta\ ADE,\quad \tan 60^0 = \frac{AE}{DE}$

$\sqrt{3} = \frac{AE}{DE}$

$AE = DE\sqrt{3}$

$AE = (16\sqrt{3})\sqrt{3}$

$AE = 16(3)$

$AE = 48\ m$

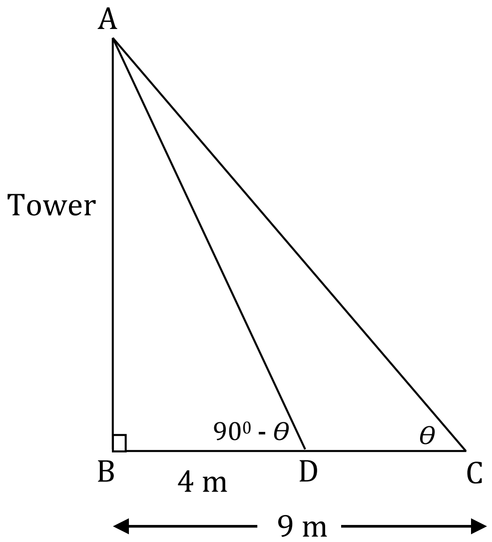
$\therefore Height\ of\ the\ hill = AE + BE$

$= 48\ m + 16\ m$

$= 64\ m$

6) The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

Solution :



$In\ \Delta\ ABC,\quad \tan\theta = \frac{AB}{BC}$

$\tan\theta = \frac{AB}{9}$

$In\ \Delta\ ADB,\quad \tan(90 - \theta) = \frac{AB}{BD}$

$\cot\theta = \frac{AB}{4}$

$\text{Now}, \tan\theta \times \cot\theta = 1$

$\frac{AB}{4} \times \frac{AB}{9} = 1$

$\frac{AB^2}{36} = 1$

$$AB^2 = 36$$

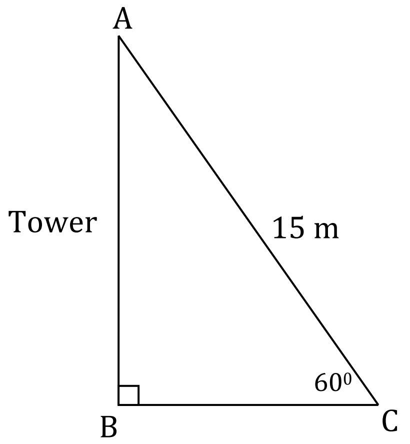
$$AB = \sqrt{36}$$

$$AB = 6\,m$$

$\therefore$ *Height of the tower is* $6\,m$.

7) **A tower stands vertically on the ground. From a point on the ground which is 15 m away from the top of the tower, the angle of elevation of the top of the tower is found to be 60^0. Find the height of the tower.**

Solution :



$$In\ \Delta\,ABC, \quad \sin 60^0 = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{15}$$

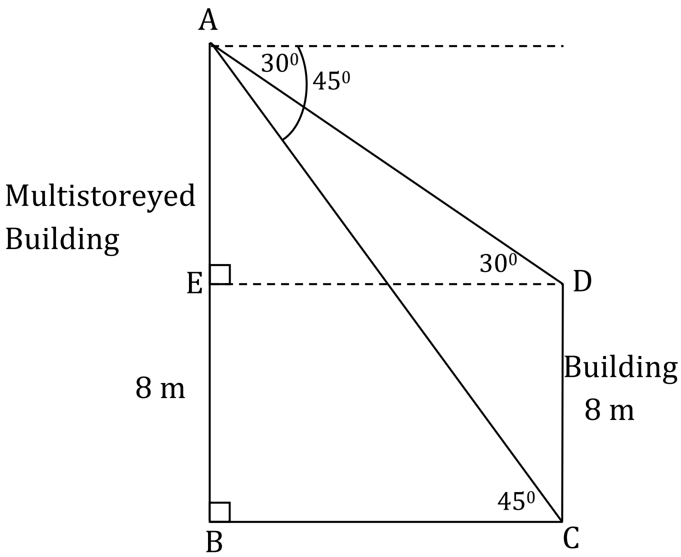
$$2AB = 15\sqrt{3}$$

$$AB = \frac{15\sqrt{3}}{2}\ m$$

$$\therefore Height\ of\ the\ tower = \frac{15\sqrt{3}}{2}\ m$$

8) **The angles of depression of the top and he bottom of an 8 m tall building from the top of a multi storeyed building are 30^0 and 45^0 respectively. Find the height of the multi-storeyed building and the distance between the two buildings.**

Solution :



$$In\ \Delta\,ABC, \quad \tan 45^0 = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC}$$

$$AB = BC$$

$$In\ \Delta\,ADE, \quad \tan 30^0 = \frac{AE}{DE}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{DE}$$

$$DE = AE\sqrt{3}\ m$$

$$BC = AE\sqrt{3}$$

$$AB = AE\sqrt{3}$$

$$AE + BE = AE\sqrt{3}$$

$$AE + 8 = AE\sqrt{3}$$

$$AE\sqrt{3} - AE = 8$$

$$AE\left(\sqrt{3} - 1\right) = 8$$

$$AE = \frac{8}{\sqrt{3} - 1}$$

$$AE = \frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{8(\sqrt{3}+1)}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{8(\sqrt{3}+1)}{3-1}$$

$$AE = \frac{8(\sqrt{3}+1)}{2}$$

$$AE = 4(\sqrt{3}+1)$$

$$AE = 4\sqrt{3}+4$$

$\therefore$ *Height of the multistoreyed building*

$$= AE + BE$$

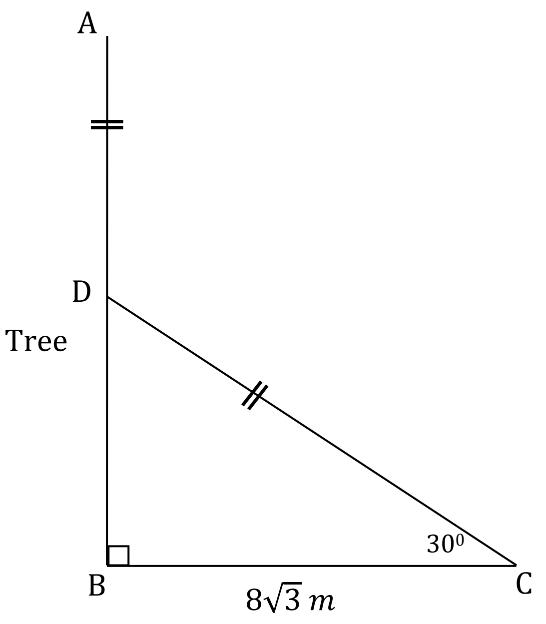
$$= 4\sqrt{3}+4+8$$

$$= (12+4\sqrt{3})$$

$$= 4(3+\sqrt{3})m$$

9) A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30⁰ with it. The distance between the foot of the tree to the point where the top touches the ground is $8\sqrt{3}$ m. Find the height of the tree.

Solution :



$$In\ \Delta DBC, \quad \tan 30^0 = \frac{DB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{DB}{8\sqrt{3}}$$

$$DB\sqrt{3} = 8\sqrt{3}$$

$$DB = \frac{8\sqrt{3}}{\sqrt{3}}$$

$$DB = 8\,m$$

$$In\ \Delta DBC, \quad \sin 30^0 = \frac{DB}{DC}$$

$$\frac{1}{2} = \frac{8}{DC}$$

$$DC = 8 \times 2$$

$$DC = 16\ m$$

$\therefore$ *Height of the tree* $= AD + DB$

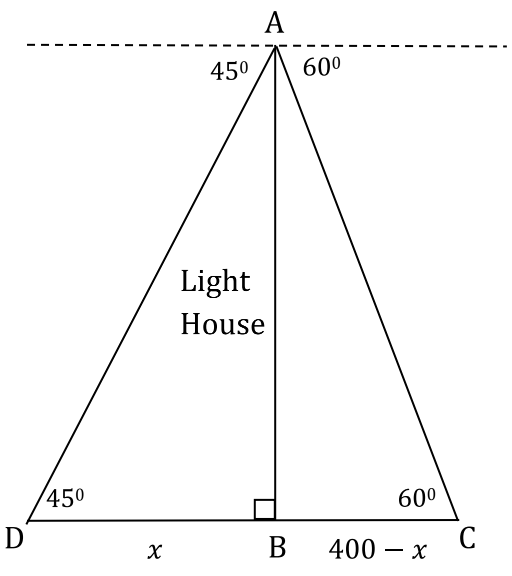
$$= DC + 8$$

$$= 16 + 8$$

$$= 24\,m$$

10) From the top of a light house, angles of depression of two ships are 45⁰ and 60⁰. The ships are on the opposite sides of the light house and in line with its foot. If the distance between the ships is 400 m, find the height of the light house.

Solution :



$$In\ \Delta\ ABD,\quad \tan 45^0 = \frac{AB}{DB}$$

$$1 = \frac{AB}{x}$$

$$AB = x$$

$$In\ \Delta\ ABC,\quad \tan 60^0 = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{x}{400 - x}$$

$$(400 - x)\sqrt{3} = x$$

$$400\sqrt{3} - x\sqrt{3} = x$$

$$x + x\sqrt{3} = 400\sqrt{3}$$

$$x(1 + \sqrt{3}) = 400\sqrt{3}$$

$$x = \frac{400\sqrt{3}}{\sqrt{3} + 1}$$

$$x = \frac{400\sqrt{3}}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$x = \frac{400\sqrt{3}(\sqrt{3} - 1)}{(\sqrt{3})^2 - (1)^2}$$

$$x = \frac{400\sqrt{3}(\sqrt{3} - 1)}{3 - 1}$$

$$x = \frac{400\sqrt{3}(\sqrt{3} - 1)}{2}$$

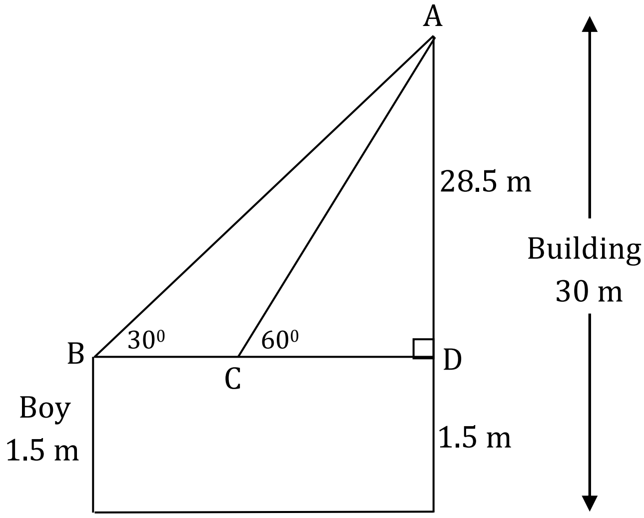
$$x = 200\sqrt{3}(\sqrt{3} - 1)$$

$$x = 200(3 - \sqrt{3})$$

$\therefore$ *Height of light house* $= 200(3 - \sqrt{3})m$

11) A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30⁰ to 60⁰, as he walks towards the building. Find the distance he walked towards the building.

Solution :



$$In\ \Delta\ ABD,\quad \tan 30^0 = \frac{AD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{28.5}{BD}$$

$$BD = 28.5\sqrt{3}\ m$$

$$In\ \Delta\ ADC,\quad \tan 60^0 = \frac{AD}{DC}$$

$$\sqrt{3} = \frac{28.5}{DC}$$

$$DC\sqrt{3} = 28.5$$

$$DC = \frac{28.5}{\sqrt{3}}$$

$$= \frac{28.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{28.5\sqrt{3}}{3}$$

$$= 9.5\sqrt{3}\ m$$

$\therefore$ *distance walked* $= BC$

$$= BD - DC$$

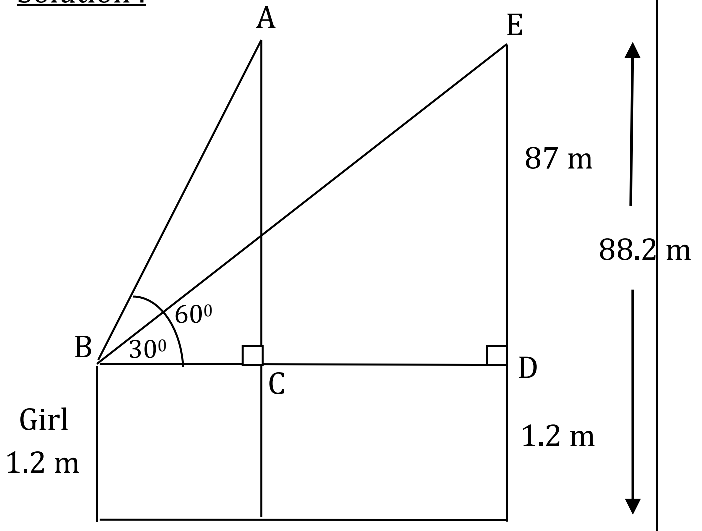
$$= 28.5\sqrt{3} - 9.5\sqrt{3}\ m.$$

$$= 19\sqrt{3}\ m$$

12) A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 600. After some time,

the angle of elevation reduces to 300. Find the distance travelled by the balloon during the interval.

Solution :



$$In\ \Delta\ EBD, \quad \tan 30^0 = \frac{ED}{BD}$$

$$\frac{\mathbf{1}}{\sqrt{3}} = \frac{87}{BD}$$

$$BD = 87\sqrt{3}\ m$$

$$In\ \Delta\ ABC, \quad \tan 60^0 = \frac{AC}{BC}$$

$$\sqrt{3} = \frac{87}{BC}$$

$$BC \times \sqrt{3} = 87$$

$$BC = \frac{87}{\sqrt{3}}$$

$$= \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{87\sqrt{3}}{3}$$

$$= 29\sqrt{3}\ m$$

$Distance\ travelled\ by\ the\ balloon$

$$= DC$$

$$= BD - BC$$

$$= 87\sqrt{3} - 29\sqrt{3}$$

$$= 58\sqrt{3}\ m$$

Lesson-13. Statistics.

Statistics is the collection, organization, analysis and interpretation of numerical data.

Ungrouped data :

If the observations are written individually, the numerical data is called ungrouped data.

Grouped data :

If the observations are written in tabular form using class-intervals and their frequencies, then the numerical data is called grouped data.

Measures of central tendency :

There are three measures of central tendency.

1) Mean.

2) Median.

3) Mode.

Arithmetic mean :

Mean or arithmetic mean of ungrouped data is given by,

$$Mean = \frac{\sum x_i}{n}$$

Where, x_i are observations and n is the number of observations.

Mean for grouped data is given by,

$$Mean = \frac{\sum f_i x_i}{\sum f_i}$$

Where, x_i are observations and f_i are their frequencies.

Mode :

Mode is the most repeated observation in the distribution of data.

Mode for ungrouped data is given by,

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where,

l = lower limit of the modal class.

h = size of the class intervals.

f_1 = frequency of the modal class.

f_0 = frequency of the class preceding the modal class.

f_2 = frequency of the class succeeding the modal class.

Median :

Median is the middle most observation in the frequency distribution.

Median for grouped data is given by,

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where,

l = lower limit of the median class.

n = number of observations.

cf = cumulative frequency of the class preceding the median class.

f = frequency of the median class.

The empirical relationship between the mean, median and mode is,

3 Median = Mode + 2 Mean.

Problems :

1) **Find the median of the following data :**

Class-Interval	Frequency
50 – 60	12
60 – 70	14
70 – 80	8
80 – 90	6
90 – 100	10

Solution :

Class-Interval	Frequency	Cf
50 – 60	12	12
60 – 70	14	26
70 – 80	8	34
80 – 90	6	40
90 – 100	10	50

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 60 + \left(\frac{\frac{50}{2} - 12}{14}\right) \times 10$$

$$= 60 + \left(\frac{25 - 12}{14}\right) \times 10$$

$$= 60 + \frac{13}{14} \times 10$$

$$= 60 + \frac{130}{14}$$

$$= 60 + 9.28$$

$$= 69.28$$

2) **Find the mode of the following distribution.**

Marks	Number of students
0 – 10	20
10 – 20	24
20 – 30	40
30 – 40	36
40 – 50	20

Solution :

Marks	Number of students
0 – 10	20
10 – 20	24
20 – 30	40
30 – 40	36
40 – 50	20

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 20 + \left(\frac{40 - 24}{2 \times 40 - 24 - 36}\right) \times 10$$

$$= 20 + \left(\frac{16}{80 - 60}\right) \times 10$$

$$= 20 + \left(\frac{16}{20}\right) \times 10$$

$$= 20 + \left(\frac{16}{2}\right)$$

$$= 20 + 8$$

$$= 28$$

3) The following table shows the age of the patients admitted in a hospital during a year. Find the mean of the given table.

Age (in years)	**Number of patients**
5 – 15	**6**
15 – 25	**11**
25 – 35	**21**
35 – 45	**23**
45 – 55	**14**
55 – 65	**5**

Solution :

CI	f_i	x_i	$f_i x_i$
5 – 15	6	10	60
15 – 25	11	20	220
25 – 35	21	30	630
35 – 45	23	40	920
45 – 55	14	50	700
55 – 65	5	60	300
	80		2830

$$Mean = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{2830}{80}$$

$$= \frac{283}{8}$$

$$= 35.37$$

4) The following frequency distribution gives the monthly consumption of electricity of 68 consumers in a locality. Find the median of the data.

Monthly consumption (units)	**Number of consumers**
65 – 85	**4**
85 – 105	**5**
105 – 125	**13**
125 – 145	**20**
145 – 165	**14**
165 – 185	**8**
185 – 205	**4**

Solution :

Monthly consumption (units)	Number of consumers f	Cf
65 – 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	8	64
185 – 205	4	68

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 125 + \left(\frac{\frac{68}{2} - 22}{20}\right) \times 20$$

$$= 125 + \left(\frac{34 - 22}{20}\right) \times 20$$

$$= 125 + \frac{12}{20} \times 20$$

$$= 125 + 12$$

$$= 137$$

5) **The marks obtained by 30 students of class X of a certain school in a mathematics paper are presented in a table below. Find the mode of the data.**

Class interval	Frequency
10 – 25	2
25 – 40	3
40 – 55	7
55 – 70	6
70 – 85	6
85 – 100	6

Solution :

Class interval	Frequency
10 – 25	2
25 – 40	3
40 – 55	7
55 – 70	6
70 – 85	6
85 – 100	6

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{7 - 3}{2 \times 7 - 3 - 6}\right) \times 15$$

$$= 40 + \left(\frac{4}{14 - 9}\right) \times 15$$

$$= 40 + \left(\frac{4}{5}\right) \times 15$$

$$= 40 + (4 \times 3)$$

$$= 40 + 12$$

$$= 52$$

6) **Find the median for the given distribution.**

Class interval	Frequency
1 – 4	6
4 – 7	30
7 – 10	40
10 – 13	16
13 – 16	4
16 – 19	4

Solution :

Class interval	Frequency	Cf
1 – 4	6	6
4 – 7	30	36
7 – 10	40	76
10 – 13	16	92
13 – 16	4	96
16 – 19	4	100

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 7 + \left(\frac{\frac{100}{2} - 36}{40}\right) \times 3$$

$$= 7 + \left(\frac{50 - 36}{40}\right) \times 3$$

$$= 7 + \frac{14}{40} \times 3$$

$$= 7 + \frac{42}{40}$$

$$= 7 + 1.05$$

$$= 8.05$$

7) The distribution below gives the weight of 30 students of a class. Find the median weight of the students.

Weight (in kg)	Number of students
40 – 45	2
45 – 50	3
50 – 55	8
55 – 60	6
60 – 65	6
65 – 70	3
70 – 75	2

Solution :

Weight (in kg)	Number of students(f)	Cf
40 – 45	2	2
45 – 50	3	5
50 – 55	8	13
55 – 60	6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 55 + \left(\frac{\frac{30}{2} - 13}{6}\right) \times 5$$

$$= 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

$$= 55 + \frac{2}{6} \times 5$$

$$= 55 + \frac{10}{6}$$

$$= 55 + 1.66 \ldots$$

$$= 56.666 \ldots$$

8) Find the mean for the given data.

Class interval	Frequency
0 – 10	3
10 – 20	5
20 – 30	9
30 – 40	5
40 – 50	3

Solution :

Class interval	Frequency f_i	x_i	$f_i x_i$
0 – 10	3	5	15
10 – 20	5	15	75
20 – 30	9	25	220
30 – 40	5	35	175
40 – 50	3	45	135
	25		620

$$Mean = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{620}{25}$$

$$= 24.8$$

Ogive curve :

It is a graphical representation of a frequency distribution, where class intervals are represented on x-axis and their corresponding cumulative frequencies are represented on y-axis.

The ogive in which upper limits are represented on x-axis and their corresponding cumulative frequencies are represented on y-axis is called **less than type ogive**.

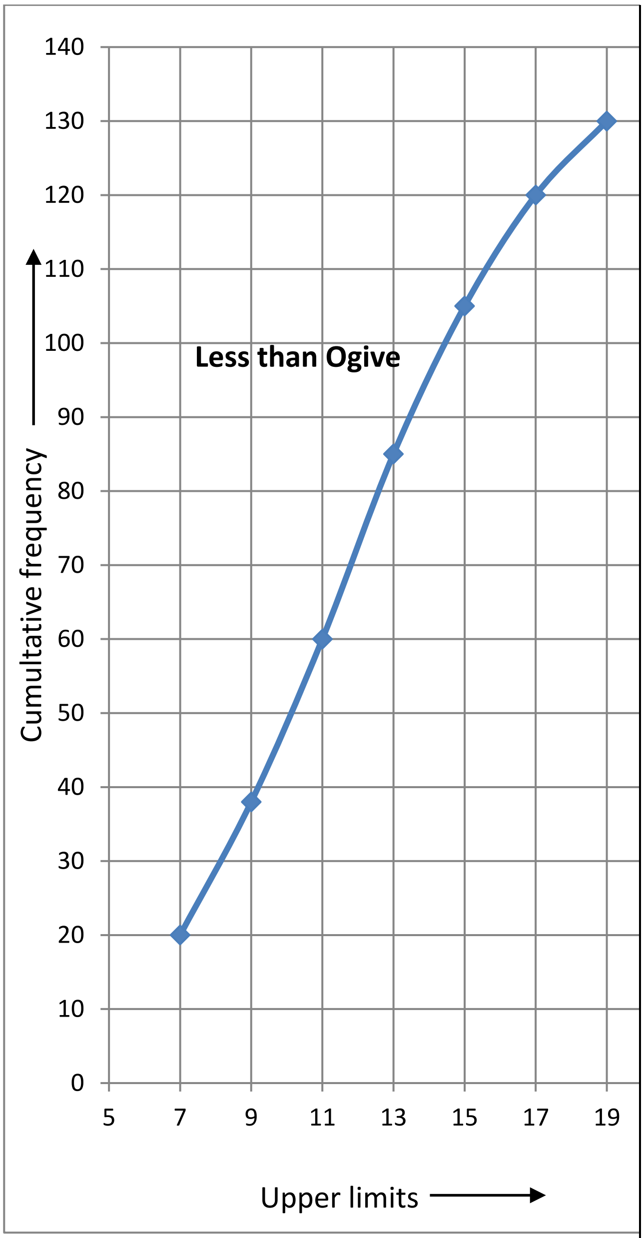
The ogive in which lower limits are represented on x-axis and their corresponding cumulative frequencies are represented on y-axis, is called **more than type ogive.**

1) **Draw less than type ogive for the given distribution and find median age of the student through the graph.**

Age of students	**Number of students**
5 – 7	**20**
7 – 9	**18**
9 – 11	**22**
11 – 13	**25**
13 – 15	**20**
15 – 17	**15**
17 – 19	**10**

Solution :

Age of students	No. of students
Less than 7	20
Less than 9	38
Less than 11	60
Less than 13	85
Less than 15	105
Less than 17	120
Less than 19	130

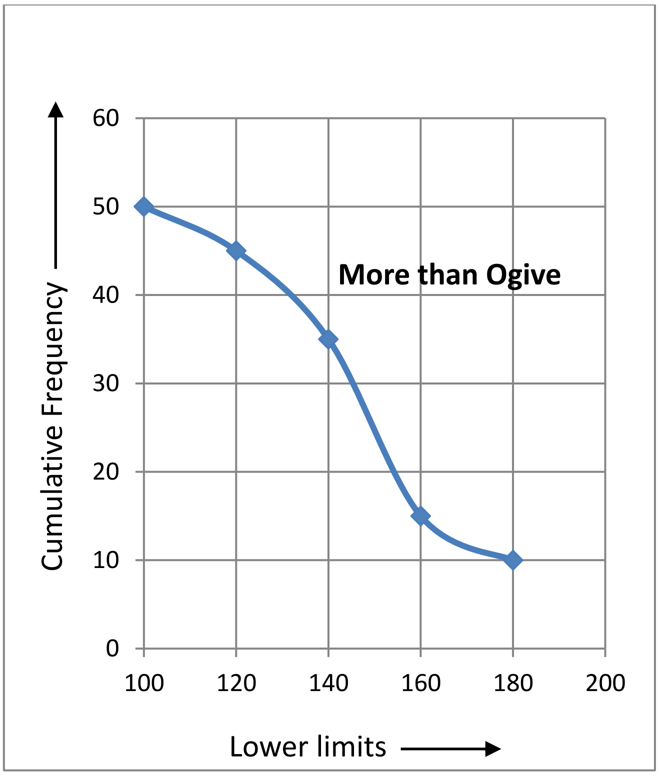


2) The following distribution gives the daily income of 50 workers of a factory. Draw its more than type ogive.

Daily income (in Rs)	Number of workers
100 – 120	5
120 – 140	10
140 – 160	20
160 – 180	5
180 – 200	10

Solution :

Daily income (in Rs)	No. of workers
More than 100	50
More than 120	45
More than 140	35
More than 160	15
More than 180	10

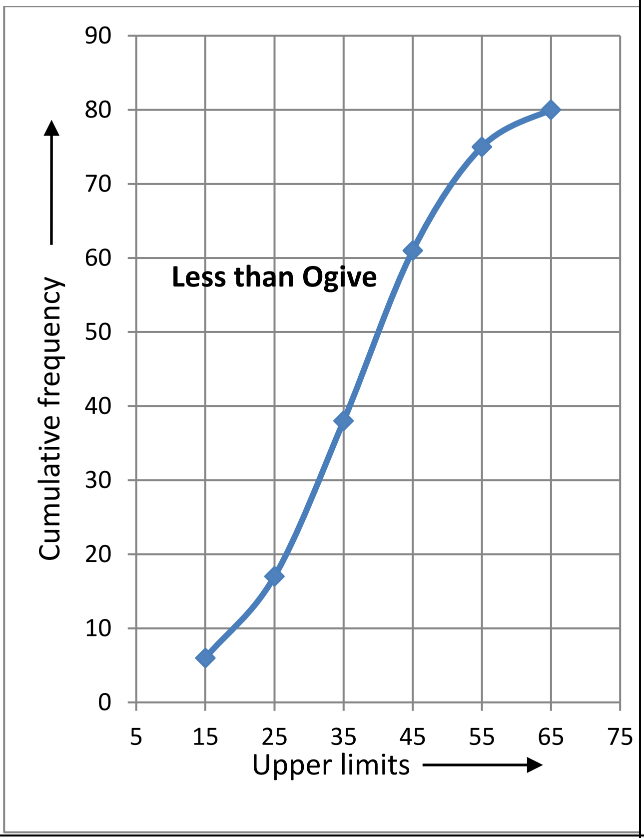


3) Draw ogive for the following data.

Class interval	Frequency
5 – 15	6
15 – 25	11
25 – 35	21
35 – 45	23
45 – 55	14
55 – 65	5

Solution :

Class interval	Frequency	Cf
Less than 15	6	6
Less than 25	11	17
Less than 35	21	38
Less than 45	23	61
Less than 55	14	75
Less than 65	5	80

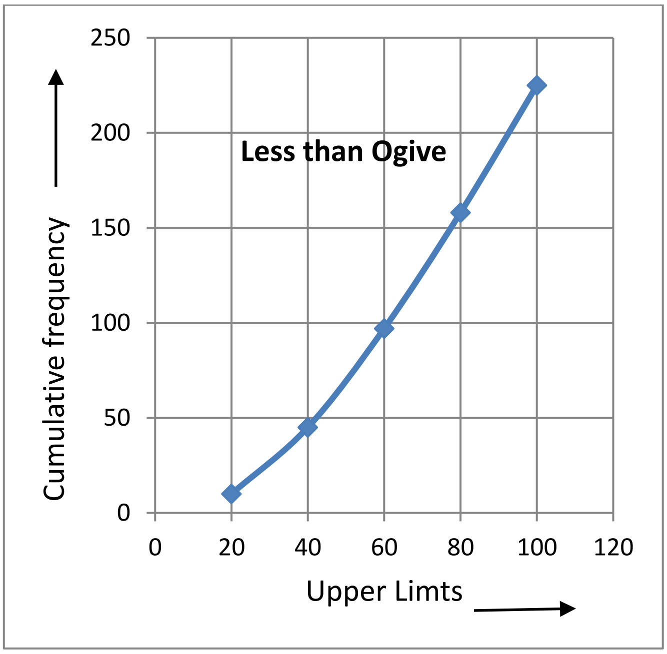


4) **The following data gives the information on the observed lifetimes (in hours) of 225 electrical components. Represent in the less than ogive.**

Life time (in hour)	**Frequency**
0 – 20	**10**
20 – 40	**35**
40 – 60	**52**
60 – 80	**61**
80 – 100	**67**

Solution :

Life time (in hour)	Frequency	Cf
Less than 20	10	10
Less than 40	35	45
Less than 60	52	97
Less than 80	61	158
Less than 100	67	225

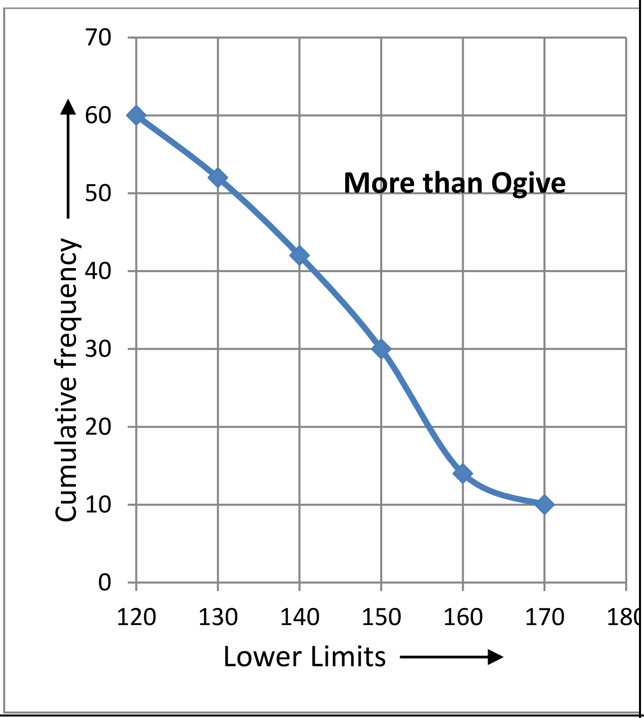


5) **Draw more than ogive for the given data.**

Heights (in cm)	**Number of students**
120 – 130	**5**
130 – 140	**10**
140 – 150	**12**
150 – 160	**16**
160 – 170	**4**
170 – 180	**10**

Solution :

Heights (in cm)	Number of students	Cf
More than 120	8	60
More than 130	10	52
More than 140	12	42
More than 150	16	30
More than 160	4	14
More than 170	10	10

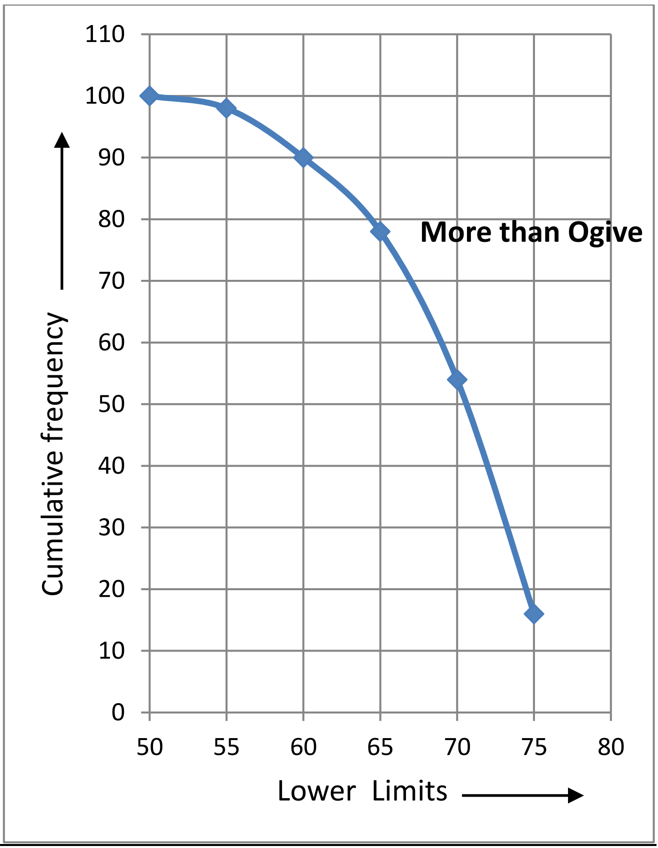


6) The following table gives production yield per hectare of wheat of 100 farms of village. Change the distribution to more than type distribution and draw its ogive.

Production yield	**Number of farms**
50 – 55	**2**
55 – 60	**8**
60 – 65	**12**
65 – 70	**24**
70 – 75	**38**
75 – 80	**16**

Solution :

Production yield	Number of farms	Cf
More than 50	2	100
More than 55	8	98
More than 60	12	90
More than 65	24	78
More than 70	38	54
More than 75	16	16



Lesson – 14. Probability

Random experiment :

An action, the results of which cannot be predicted is called a random experiment.

Sample space :

The collection of all the outcomes of a random experiment is called sample space.

Event :

An action that has particular outcomes of a random experiment is called event.

Probability :

Probability of an event tells the chance of happening of an event.

$$Probability = \frac{No. of\ favourable\ outcomes}{No. of\ possible\ outcomes}$$

$$P(E) = \frac{n(E)}{n(S)}$$

The probability of an event lies between 0 and 1. That is, $0 \leq P(E) \leq 1$.

Elementary events :

An event having only one outcome of a random experiment is called an elementary event.

The sum of the probabilities of all the elementary events is 1.

Complementary events :

If E is an event, then 'Not E' is called complementary event of E.

$P(E) + P(\overline{E}) = 1$

Impossible event :

An event which cannot happen is called an impossible event and its probability is 0.

Certain event :

An event which is certain to happen is called sure or certain event and its probability is 1.

Playing cards :

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each.

1) Spades (Black colour)
2) Diamonds (Red colour)
3) Hearts (Red colour)
4) Clubs (Black colour)

The cards in each suit are

Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, 2. Kings, Queens and Jacks are called face cards.

Problems :

1) Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

Solution :

S = {H, T}

$$P(head) = \frac{1}{2}$$

$$P(tail) = \frac{1}{2}$$

2) A die is thrown once, find the probability of getting,

i) a prime number. ii) a square number.

Solution :

S = {1, 2, 3, 4, 5, 6}

$i)\ A = \{2, 3, 5\}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

$ii)\ B = \{1, 4\}$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

3) Suppose a die is thrown once. What is the probability of getting a number,

(i) Greater than 4.

(ii) Less than or equal to 4.

Solution :

$S = \{1, 2, 3, 4, 5, 6\}$

$i)\ A = \{5, 6\}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

$ii)\ B = \{1, 2, 3, 4\}$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

4) Two unbiased dice are rolled once. What is the probability of getting,

i) A doublet.

ii) A sum equal to 7.

Solution :

$S = \{(1, 1), (1, 2), (1, 3), (1, 4) \ldots\ldots (6, 6)\}$

(i) $A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) $B = \{(1,6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

5) If two dice are thrown once. Find the probability of getting the sum of the digits on the faces of the die is 8.

Solution :

$S = \{(1, 1), (1, 2), (1, 3), (1, 4) \ldots\ldots (6, 6)\}$

$A = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

6) A box contains 5 red marbles, 3 white marbles and 6 green marbles. One marble is taken out of the box at random, what is the probability that the marble taken out will not be green?

Solution :

Red balls = 5

White balls = 3

Green balls = 6

Total balls = 14

$$P(not\ green) = \frac{8}{14} = \frac{4}{7}$$

7) A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag.

What is the probability that the ball drawn is,

(i) Red?

(ii) Not red?

Solution :

Red balls = 3

Black balls = 5

Total balls = 8

$$P(Red) = \frac{3}{8}$$

$$P(Not\ red) = \frac{5}{8}$$

8) A box contains 28 bulbs of which 7 bulbs are defective, a bulb is drawn randomly from the box. Find the probability of picking a non-defective bulb?

Solution :

Total bulbs = 28

Defective bulbs = 7

Non – defective bulbs = 21

$P(Non-defective) = \frac{21}{28} = \frac{3}{4}$

9) A lot of 25 bulbs contain 5 defective ones. One bulb is drawn at random from the lot. What is the probability that the bulb is good?

Solution :

Total bulbs = 25

Defective ones = 5

Good ones = 20

$P(good\ ones) = \frac{20}{25}$

$= \frac{4}{5}$

10)If A is an event in a random experiment such that P(A̅) : P(A) = 5 : 11, then find P(Ā) and P(A).

Solution :

P(A) : P(A̅) = 5 : 11

Sum = 16

$P(A) = \frac{5}{16}$

$P(A^{-}) = \frac{11}{16}$

11)Two dice are thrown simultaneously. Find the probability of getting,

a) same number on both faces.

b) both faces having multiples of 5.

Solution :

S = {(1, 1), (1, 2), (1, 3) ....(6, 6)}

(a) A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6,6)}

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{6}{36}$

$= \frac{1}{6}$

(b) B = {(5, 5)}

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{36}$

12)A die numbered 1 to 6 on its faces is rolled once. Find the probability of getting either an even number or a multiple of 3 on its top face.

Solution :

S = {1, 2, 3, 4, 5, 6}

A = {2, 3, 4, 6}

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

13)A cubical die numbered from 1 to 6 is rolled twice. Find the probability of getting the sum of numbers on its faces is 10.

Solution :

S = {(1, 1), (1, 2), (1, 3) ....(6, 6)}

A = {(4, 6), (5, 5), (6, 4)}

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{3}{36} = \frac{1}{12}$

14)A die is thrown twice. What is the probability that,

i) 5 will not come up either time.

ii) 5 will come up at least once.

S = {(1, 1), (1, 2), (1, 3) ....(6, 6)}

(i) A = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 6),
(4, 1), (4, 2),(4, 3), (4,4), (4,6),
(6,1), (6,2), (6,3), (6,4), (6,5)}

$P(A) = \frac{n(A)}{n(S)} = \frac{25}{36}$

(ii) B = {(1, 5),(2, 5),(3, 5),(4, 5),(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 5)}

$$P(B) = \frac{n(B)}{n(S)} = \frac{11}{36}$$

15)Two dice, one blue and one grey, are thrown at same time. What is the probability that

i) Sum is 8?

ii) Sum is 13?

iii) Less than or equal to 12?

Solution :

S = {(1, 1), (1, 2), (1, 3), (1, 4) ...... (6, 6)}

(i) A = {(2, 6), (3, 5), (4,4), (5, 3), (6, 2)}

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

(ii) B = { }

$$P(B) = \frac{n(B)}{n(S)} = \frac{0}{36} = 0$$

(ii) C = {(1, 1), (1, 2), (1, 3), (1, 4), .. (6, 6)}

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$$

16)Harpreet tosses two different coins simultaneously. What is the probability that she gets at least one head?

Solution :

S = {HH, HT, TH, TT}

$$P(at\ least\ one\ head) = \frac{3}{4}$$

17)Three fair coins are tossed simultaneously; find the probability of getting minimum one tail.

Solution :

S = {HHH, HTH, HHT, HTT, THH,THT,TTH,TTT}

$$P(minimum\ one\ tail) = \frac{7}{8}$$

18)One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting,

i) A face card.

ii) A spade.

Solution :

Total cards = 52

(i) Face cards = 4

$$P(face\ card) = \frac{4}{52} = \frac{1}{13}$$

(ii) Spades = 13

$$P(a\ spade) = \frac{13}{52} = \frac{1}{4}$$

19)Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?

Probability of winning = 0.62

Probability of losing = 1 – 0.62

= 0.38

20) If P(E) = 0.05, what is the probability of 'not E'?

P(not E) = 1 – P(E)

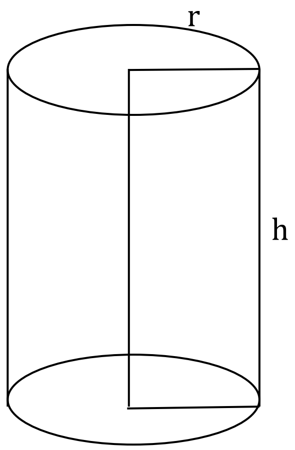
= 1 – 0.05

= 0.95

Lesson – 15.

Surface Area and Volume

Cylinder :



A cylinder is a solid described by the rotation of a rectangle about one of its side as axis.

A cylinder has two plane or flat surfaces and one curved surface. The plane surfaces are circular in shape and are called circular bases.

'r' is the radius of the circular base and 'h' is the height of the cylinder.

$Curved\ surface\ area\ of\ cylinder\ =\ 2\pi rh$

$Total\ surface\ area\ of\ cylinder$

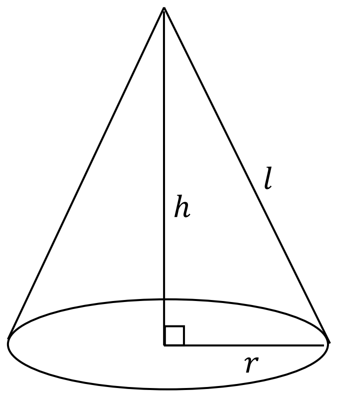
$=\pi r^2 + \pi r^2 + 2\pi rh$

$= 2\pi r^2 + 2\pi rh$

$= 2\pi r(r+h)$

$Volume\ of\ cylinder = \pi r^2 h$

Cone :



A cone is a solid described by the rotation of a right angled triangle about one its sides containing the right angle as axis.

A cone has one plane or flat surface and one curved surface. The plane surface is circular in shape and is the circular base of the cone.

'r' is the radius of the circular base, 'h' is the height of the cone and 'l' is the slant height of the cone.

$CSA\ of\ cone\ =\ \pi rl$

$TSA\ of\ cone\ = \pi r^2 + \pi rl$

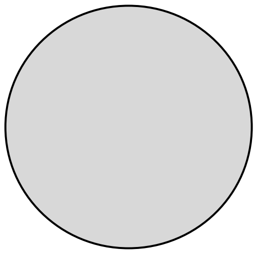
$= \pi r(r+l)$

$Volume\ of\ cone = \frac{1}{3}\pi r^2 h$

$l^2 = r^2 + h^2$

Sphere :

A sphere is a solid described by the rotation of a semicircle about its diameter as axis.

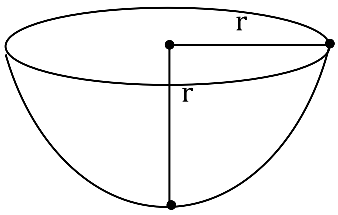


$Surface\ area\ of\ a\ sphere = 4\pi r^2$

$Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$

Hemisphere :

A plane through the centre of a sphere divides it into two equal parts, each is called a hemisphere.



A hemisphere has a plane circular surface and a curved surface.

$Plane\ surface\ area\ of\ a\ hemisphere = \pi r^2$

$Curved\ surface\ area\ of\ hemisphere = 2\pi r^2$

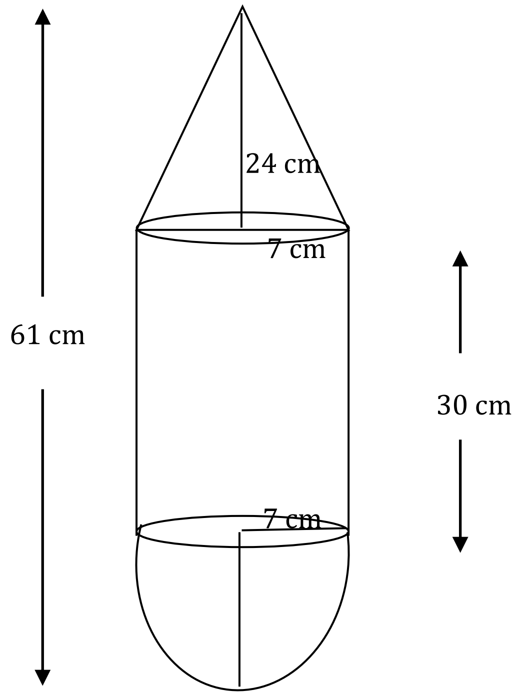
$Total\ surface\ area\ of\ a\ hemisphere = 3\pi r^2$

$Volume\ of\ a\ hemisphere = \frac{2}{3}\pi r^3$

Problems :

1) **A cone and a hemisphere are joined on either sides of a cylinder. These solids have radius 7 cm each. If the total height of the solid is 61 cm and the height of the cylinder is 30 cm, find the cost of painting the outer surface of the solid at the rate of Rs. 10 per 100 cm².**

Solution :



$l^2 = r^2 + h^2$

$= 7^2 + 24^2$

$= 49 + 576$

$= 625$

$l = \sqrt{625}$

$l = 25\ cm$

$CSA\ of\ cone = \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 550\ cm^2$

$CSA\ of\ cylinder = 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 30$

$= 1320\ cm^2$

$CSA\ of\ hemisphere = 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 7$

$= 308\ cm^2$

$Outer\ surface\ of\ the\ solid$

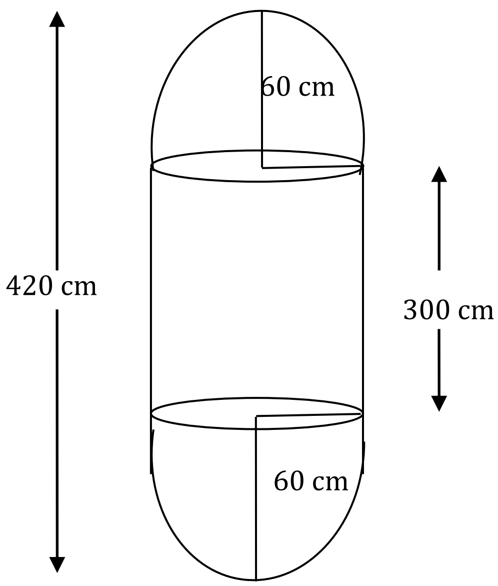
$= 550\ cm^2 + 1320\ cm^2 + 308\ cm^2$

$= 2178\ cm^2$

$$Cost\ of\ painting = Rs.\frac{10}{100} \times 2178$$
$$= Rs. 217.8$$
$$= Rs. 218.$$

2) **A social welfare association decides to supply drinking water for the flood affected people. The drinking water is filled in a water tanker which is the shape of a cylinder with hemispherical ends. The whole length of the tanker is 4.2 m and the diameter of the base of the cylinder and two hemispheres are each 1.2 m. If they distribute drinking water to 60 people in a container, each is in the shape of a cylinder of radius 21 cm and height 50 cm, find the quantity of water left in the tanker after distribution in litres.**

Solution :



$Volume\ of\ 2\ hemispheres$
$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \pi \times 60 \times 60 \times 60$$
$$= 288000\pi\ cm^3$$

$Volume\ of\ cylinder$
$$= \pi r^2 h$$
$$= \pi \times 60 \times 60 \times 300$$
$$= 1080000\pi\ cm^3$$

$Total\ volume\ of\ the\ tank$
$$= 288000\ \pi cm^3 + 1080000\pi\ cm^3$$
$$= 1368000\ \pi\ cm^3$$

$Volume\ of\ container$
$$= \pi r^2 h$$
$$= \pi \times 21 \times 21 \times 50$$
$$= 22050\pi\ cm^3$$

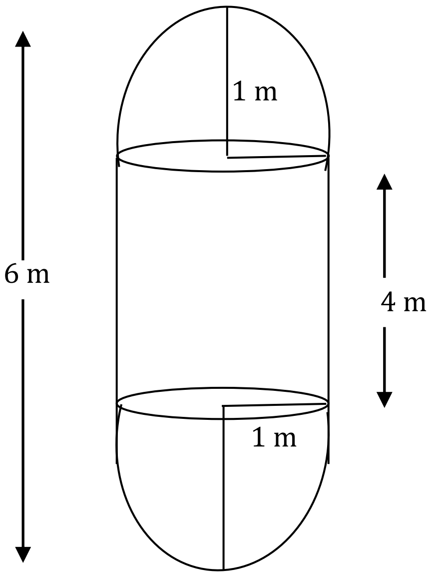
$Quantity\ of\ water\ distributed$
$$= 60 \times 22050\pi\ cm^3$$
$$= 1323000\pi\ cm^3$$

$Quantity\ of\ water\ left\ in\ the\ tank$
$$= 1368000\pi\ cm^3 - 1323000\pi\ cm^3$$
$$= 45000\pi\ cm^3$$
$$= 45\pi\ litres$$
$$= 45 \times 3.14$$
$$= 141.3\ litres$$

3) **A milk tank is in the shape of cylinder with hemisphere of same radius attached to both ends. If the total height of the tank is 6 m and the radius is 1 m. Calculate the maximum quantity of milk filled in the tank in litres. And also**

calculate the total surface area of the tank.

Solution :



$Volume\ of\ 2\ hemispheres$

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 1 \times 1 \times 1$

$= \frac{4\pi}{3}\ \ m^3$

$Volume\ of\ cylinder$

$= \pi r^2 h$

$= \pi \times 1 \times 1 \times 4$

$= 4\pi\ \ m^3$

$Total\ volume\ of\ the\ tank$

$= \frac{4\pi}{3}\ + 4\pi$

$= \frac{4\pi + 12\pi}{3}$

$= \frac{16\pi}{3}\ m^3$

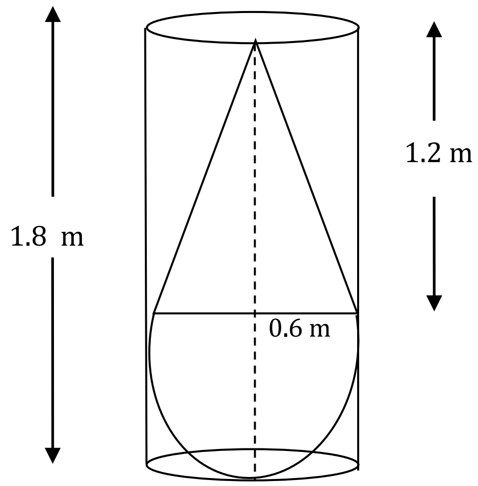
$= \frac{16}{3} \times 3.14\ m^3$

$\approx 16\ m^3$

$\approx 16{,}000\ litres$

4) A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder in m³, if the radius of the cylinder is 60 cm and its height is 180 cm.

Solution :



$Volume\ of\ \ hemisphere$

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 0.6 \times 0.6 \times 0.6$

$= 0.144\ \pi\ m^3$

$Volume\ of\ cone$

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 0.6 \times 0.6 \times 1.2$

$= 0.144\pi\ m^3$

$Volume\ of\ cylinder$

$= \pi r^2 h$

$= \pi \times 0.6 \times 0.6 \times 1.8$

$= 0.648\pi\ m^3$

$Volume\ of\ water\ left\ in\ the\ cylinder$

$= 0.648\pi - (0.144\pi + 0.144\pi)$

$= 0.648\pi - 0.288\pi$

$= 0.36\,\pi\ m^3$

$= 0.36 \times 3.14$

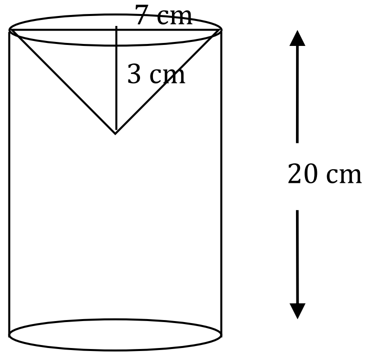
$= 1.1304\ m^3$

$= 1.1304 \times 1000\ l$

$= 1130.4\ litres$

5) The bottom of a right cylindrical shaped vessel made from metallic sheet is closed by a cone shaped vessel. The radius of the circular base of the cylinder and radius of the base of the cone each is equal to 7 cm. If the height of the cylinder is 20 cm and height of the cone is 3 cm. Calculate the cost of the milk to fill completely this vessel at the rate of Rs. 20 per litre.

Solution :



$Volume\ of\ cylinder = \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 20$

$= 3080\ cm^3$

$Volume\ of\ cone = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 3$

$= 154\ cm^3$

$Volume\ of\ the\ vessel$

$= 3080\ cm^3 - 154\ cm^3$

$= 2926\ cm^3$

$= 2.926\ litres$

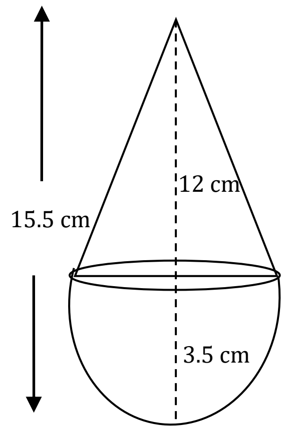
$= 3\ litres$

$Cost\ of\ the\ milk = Rs. 20 \times 3$

$= Rs. 60$

6) A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Solution :



$l^2 = r^2 + h^2$

$= (3.5)^2 + 12^2$

$= 12.25 + 144$

$= 156.25$

$l = \sqrt{156.25}$

$l = 12.5\ cm$

$CSA\ of\ cone = \pi rl$

$= \frac{22}{7} \times 3.5 \times 12.5$

$= 137.5\ cm^2$

$CSA\ of\ hemisphere = 2\pi r^2$

$= 2 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 77\ cm^2$

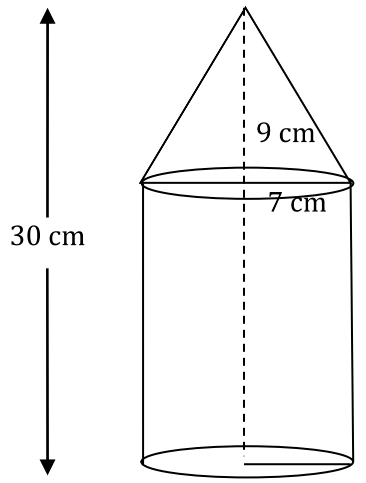
$Total\ surface\ area\ of\ the\ toy$

$= 137.5\ cm^2 + 77\ cm^2$

$= 214.5\ cm^2$

7) A solid is in the form of a cone mounted on a right circular cylinder both having same radii. The radius of the base and height of the cone are 7 cm and 9 cm respectively. If the total height of the solid is 30 cm, find the volume of the solid.

Solution :



$Volume\ of\ cone = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 9$

$= 462\ cm^3$

$Volume\ of\ cylinder = \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 21$

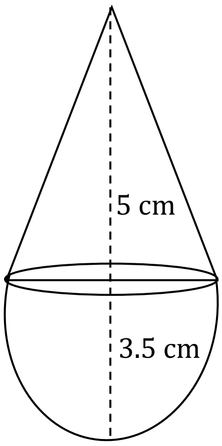
$= 3234\ cm^3$

$Volume\ of\ solid = 462\ cm^3 + 3234\ cm^3$

$= 3696\ cm^3$

8) A toy is in the form of a cone mounted on a hemisphere. If the radius of these solids is 3.5 cm and height of the cone is 5 cm, find the volume of the toy.

Solution :



$Volume\ of\ hemisphere$

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$

$= \frac{269.5}{3}\ cm^3$

$Volume\ of\ cone$

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 5$

$= \frac{192.5}{3}\ cm^3$

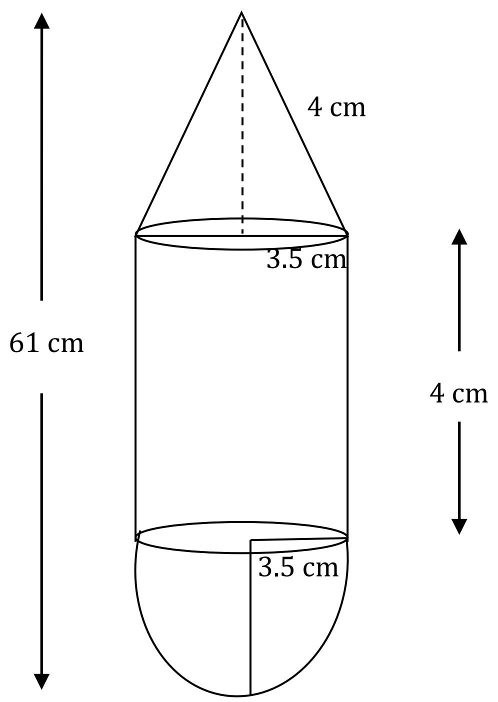
$Volume\ of\ toy = \frac{269.5}{3} + \frac{192.5}{3}$

$= \frac{462}{3}$

$= 154\ cm^3$

9) **A solid is composed of a cylinder with a hemisphere at one end and a cone at the other end. If the radius of each of the solids is 3.5 cm and height of the cylinder is equal to the slant height of the cone, find the total surface area of the solid if the slant height is 4 cm.**

Solution :



$CSA\ of\ cone = \pi rl$

$= \frac{22}{7} \times 3.5 \times 4$

$= 44\ cm^2$

$CSA\ of\ cylinder = 2\pi rh$

$= 2 \times \frac{22}{7} \times 3.5 \times 4$

$= 88\ cm^2$

$CSA\ of\ hemisphere = 2\pi r^2$

$= 2 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 154\ cm^2$

$Total\ surface\ area\ of\ the\ solid$

$= 44\ cm^2 + 88\ cm^2 + 154\ cm^2$

$= 286\ cm^2$

10) **The circumference of the base of a cylinder is 132 cm and its height is 25 cm. Find the volume of the cylinder.**

Solution :

$2\pi r = 132\ cm$

$r = \frac{132}{2\pi}$

$= \frac{132 \times 7}{2 \times 22}$

$= \frac{66 \times 7}{22}$

$= 3 \times 7$

$= 21\ cm$

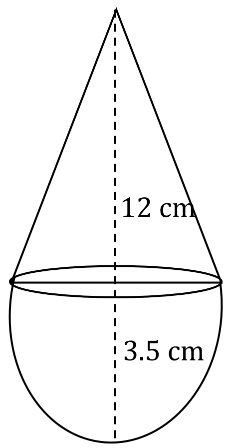
$Volume\ of\ cylinder = \pi r^2 h$

$= \frac{22}{7} \times 21 \times 21 \times 25$

$= 34650\ cm^3$

11) **A wooden solid is made by mounting a cone on a hemisphere. If the area of base of the cone is 38.5 cm² and total height of the solid is 15.5 cm, then find the total surface area and volume of the solid.**

Solution :



$Area\ of\ base = 38.5\ cm^2$

$\pi r^2 = 38.5$

$r^2 = \frac{38.5}{\pi}$

$= 38.5 \times \frac{7}{22}$

$= 12.25$

$r = \sqrt{12.25}$

$= 3.5\ cm$

$l^2 = r^2 + h^2$

$= (3.5)^2 + 12^2$

$= 12.25 + 144$

$= 156.25$

$l = \sqrt{156.25}$

$l = 12.5$

$CSA\ of\ cone = \pi rl$

$= \frac{22}{7} \times 3.5 \times 12.5$

$= 137.5\ cm^2$

$CSA\ of\ hemisphere = 2\pi r^2$

$= 2 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 77\ cm^2$

$TSA\ of\ solid = 137.5 + 77$

$= 214.5\ cm^2$

$Volume\ of\ cone = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12$

$= 154\ cm^3$

$Volume\ of\ hemisphere$

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$

$= \frac{269.5}{3}$

$= 89.83\ cm^3$

$Volume\ of\ solid = 154 + 89.83$

$= 243.83\ cm^3$

Conversion of solid from one shape to other

1) The radius of a solid metallic sphere is 10 cm. It is melted and recast into small cones of height 10 cm and base radii 5 cm. Find the number of small cones.

Solution :

$Number\ of\ cones = \frac{Volume\ of\ sphere}{Volume\ of\ each\ cone}$

$= \frac{\frac{4}{3}\pi r^3}{\frac{1}{3}\pi r^2 h}$

$= \frac{4r^3}{r^2 h}$

$= \frac{4 \times 10 \times 10 \times 10}{5 \times 5 \times 10}$

$= 16\ cones$

2) A hemispherical vessel of radius 14 cm is fully filled with sand. This sand is poured on a level ground. The heap of sand forms a cone shape of height 7 cm. Calculate the area of ground occupied by the circular base of the heap of the sand.

Solution :

$Volume\ of\ cone = Volume\ of\ hemisphere$

$$\frac{1}{3}Bh = \frac{2}{3}\pi r^3$$

$$Bh = 2\pi r^3$$

$$B \times 7 = 2 \times \frac{22}{7} \times 14 \times 14 \times 14$$

$$B \times 7 = 88 \times 14 \times 14$$

$$B = \frac{88 \times 14 \times 14}{7}$$

$$= 2464\ cm^2$$

3) A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform $22\ m \times 14\ m$. Find the height of the platform.

Solution :

$Volume\ of\ Platform = Volume\ of\ cylinder$

$$l \times b \times h = \pi r^2 h$$

$$22 \times 14 \times h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20$$

$$22 \times 14 \times h = 11 \times 7 \times 10$$

$$h = \frac{11 \times 7 \times 10}{22 \times 14}$$

$$h = \frac{10}{2 \times 2}$$

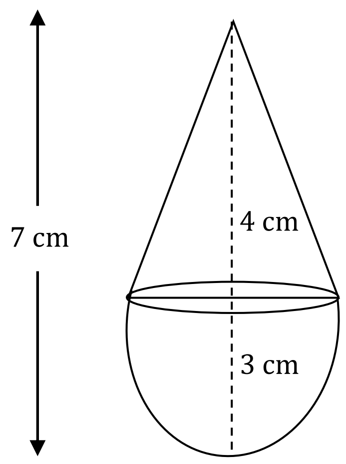
$$= \frac{10}{4}$$

$$h = 2.5\ m$$

$\therefore Height\ of\ the\ platform\ is\ 2.5\ m$

4) A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into a toy in the shape of right circular cone mounted on a hemisphere. If the radii of the cone and the hemisphere are equal to 3 cm and the height of the toy is 7 cm, calculate the number of such toys that can be formed.

Solution :



$Number\ of\ toys$

$$= \frac{Volume\ of\ cylinder}{Volume\ of\ each\ toy}$$

$$= \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3}$$

$$= \frac{6 \times 6 \times 15}{\left(\frac{1}{3} \times 3 \times 3 \times 4\right) + \left(\frac{2}{3} \times 3 \times 3 \times 3\right)}$$

$$= \frac{6 \times 6 \times 15}{(3 \times 4) + (2 \times 3 \times 3)}$$

$$= \frac{6 \times 6 \times 15}{12 + 18}$$

$$= \frac{6 \times 6 \times 15}{30}$$

$$= 18\ toys$$

5) A right circular metallic cone of height 20 cm and base radius 5 cm is melted and recast into a solid sphere. Find the radius of the sphere.

Solution :

$Volume\ of\ sphere = volume\ of\ cone$

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$4R^3 = r^2 h$$

$$4R^3 = 5 \times 5 \times 20$$

$$R^3 = \frac{5 \times 5 \times 20}{4}$$

$R^3 = 5 \times 5 \times 5$

$R^3 = 5^3$

$R = 5\ cm$

6) **A solid sphere of radius 3 cm is melted and reformed by stretching it into a cylindrical shaped wire of length 9 m. Find the radius of the wire.**

Solution :

$Volume\ of\ cylinder = volume\ of\ sphere$

$\pi r^2 h = \frac{4}{3}\pi r^3$

$r^2 h = \frac{4}{3} r^3$

$r^2 \times 900 = \frac{4}{3} \times 3 \times 3 \times 3$

$r^2 \times 900 = 4 \times 9$

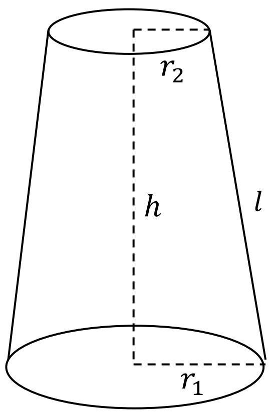
$r^2 = \frac{4 \times 9}{900}$

$r^2 = \frac{4}{100}$

$r = \sqrt{0.04}$

$r = 0.2\ cm$

Frustum of a cone :



r_1 and r_2 are the radii of the ends of the frustum of the cone. r_1 is the radius of the larger base and r_2 is the radius of the smaller base.

h is the height of the frustum of the cone.

l is the slant height of the frustum of the cone.

$Volume\ of\ the\ frustum\ of\ cone$

$= \frac{1}{3}\pi({r_1}^2 + {r_2}^2 + r_1 r_2)h$

$CSA\ of\ frustum\ of\ cone = \pi(r_1 + r_2)l$

$TSA\ of\ frustum\ of\ cone$

$= \pi(r_1 + r_2)l + \pi {r_1}^2 + \pi {r_2}^2$

$Slant\ height\ of\ frustum\ is\ given\ by$

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

Problems :

1) **A glass is in the shape of frustum of a cone of height 12 cm. The diameters of its two circular ends are 6 cm and 4 cm. Find the capacity of the glass.**

Solution :

$Volume\ of\ the\ frustum\ of\ cone$

$= \frac{1}{3}\pi({r_1}^2 + {r_2}^2 + r_1 r_2)h$

$= \frac{1}{3}\pi[(3^2 + 2^2 + (3)(2)] \times 12$

$= \pi(9 + 4 + 6) \times 4$

$= \pi(19) \times 4$

$= 76\pi\ cm^3$

2) **The radii of two circular ends of a frustum of a cone shaped dustbin are 15**

cm and 8 cm. If its depth is 63 cm, find the volume of the dustbin.

Solution :

$Volume\ of\ the\ dustbin$

$$= \frac{1}{3}\pi({r_1}^2 + {r_2}^2 + r_1r_2)h$$

$$= \frac{1}{3}\pi[(15^2 + 8^2 + (15)(8)] \times 63$$

$$= \frac{22}{7}(225 + 64 + 120) \times 21$$

$$= 22(409) \times 3$$

$$= 26{,}994\ cm^3$$

3) A drinking glass vessel is in the shape of frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass vessel.

Solution :

$Volume\ of\ the\ glass\ vessel$

$$= \frac{1}{3}\pi({r_1}^2 + {r_2}^2 + r_1r_2)h$$

$$= \frac{1}{3} \times \frac{22}{7}[(2^2 + 1^2 + 2(1)] \times 14$$

$$= \frac{1}{3} \times 22(4 + 1 + 2) \times 2$$

$$= \frac{1}{3} \times 22 \times 7 \times 2$$

$$= \frac{308}{3}$$

$$= 102.66\ cm^3$$

4) The slant height of the frustum of a cone is 4 cm and the perimeter of its circular bases are 18 cm and 6 cm respectively. Find the curved surface area and total surface area of the frustum.

Solution :

$$2\pi r_1 = 18\ cm$$

$$r_1 = \frac{18}{2\pi} = \frac{9}{\pi}$$

$$2\pi r_2 = 6\ cm$$

$$r_2 = \frac{6}{2\pi} = \frac{3}{\pi}$$

$$CSA\ of\ frustum\ of\ cone = \pi(r_1 + r_2)l$$

$$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4$$

$$= \pi\left(\frac{12}{\pi}\right) \times 4$$

$$= 48\ cm^2$$

$TSA\ of\ frustum$

$$= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$$

$$= 48 + \pi\left[\left(\frac{9}{\pi}\right)^2 + \left(\frac{3}{\pi}\right)^2\right]$$

$$= 48 + \pi\left(\frac{81}{\pi^2} + \frac{9}{\pi^2}\right)$$

$$= 48 + \pi\left(\frac{90}{\pi^2}\right)$$

$$= 48 + \frac{90}{\pi}$$

5) An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The diameters of the two circular ends of the bucket are 42 cm and 28 cm. The total vertical height of the bucket is 30 cm and that of the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket. Also find the volume of the water the bucket can hold?

Solution :

$Height\ of\ the\ bucket = 30\ cm$

$Height\ of\ the\ base = 6\ cm$

$Height\ of\ the\ frustum = 24\ cm$

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{24^2 + (21 - 14)^2}$

$= \sqrt{576 + 7^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

$= 25\ cm$

$CSA\ of\ frustum\ = \pi(r_1 + r_2)l$

$= \frac{22}{7}(21 + 14) \times 25$

$= \frac{22}{7} \times 35 \times 25$

$= 22 \times 5 \times 25$

$= 2750\ cm^2$

$CSA\ of\ cylinder = 2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 6$

$= 44 \times 2 \times 6$

$= 1848\ cm^2$

$Area\ of\ metal\ sheet = 2750 + 1848$

$= 4598\ cm^2$

$Volume\ of\ the\ bucket$

$= \frac{1}{3}\pi({r_1}^2 + {r_2}^2 + r_1 r_2)h$

$= \frac{1}{3}\pi[(21^2 + 14^2 + (21)(14)] \times 30$

$= \frac{22}{7}(441 + 196 + 294) \times 10$

$= \frac{22}{7} \times 931 \times 10$

$= 29260\ cm^3$

6) A bucket is in the shape of frustum with top and bottom of the circle of radii 15 cm and 10 cm. Its depth is 12 cm, find its curved surface area and total surface area.

Solution :

$l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{12^2 + (15 - 10)^2}$

$= \sqrt{144 + 5^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= 13\ cm$

$CSA\ of\ frustum\ = \pi(r_1 + r_2)l$

$= \frac{22}{7}(15 + 10) \times 13$

$= \frac{22}{7} \times 25 \times 13$

$= \pi \times 325$

$= 325\pi\ cm^2$

$TSA\ of\ frustum$

$= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$

$= 325\pi + \pi(15^2 + 10^2)$

$= 325\pi + \pi(225 + 100)$

$= 325\pi + 325\pi$

$= 650\pi\ cm^2$

Formulas :

Arithmetic Progressions.

1) $a_n = a + (n-1)d$

2) $S_n = \frac{n}{2}[2a + (n-1)d]$

3) $S_n = \frac{n}{2}(a + l)$

4) $S_n - S_{n-1} = a_n$

5) $Sum\ of\ first\ n\ natural\ numbers,$

$S_n = \frac{n(n+1)}{2}$

6) $Sum\ of\ first\ n\ odd\ numbers,$

$S_n = n^2$

7) $Sum\ of\ first\ n\ even\ numbers,$

$S_n = n(n+1)$

Areas related to circles.

1) $Perimeter\ of\ square = 4a$

2) $Area\ of\ square = a^2$

3) $Perimeter\ of\ rectangle = 2(l + b)$

4) $Area\ of\ rectanlge = l \times b$

5) $Perimeter\ of\ circle = 2\pi r$

6) $Area\ of\ circle = \pi r^2$

7) $Area\ of\ sector = \frac{\theta}{360} \times \pi r^2$

8) $Length\ of\ arc = \frac{\theta}{360} \times 2\pi r$

9) $Length\ of\ semiarc = \pi r$

10) $Area\ of\ semi\ circle = \frac{1}{2}\pi r^2$

Coordinate Geometry

1) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2) $P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

3) $Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

4) $Area\ of\ \Delta ABC$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Real numbers.

1) $\mathrm{H \times L = a \times b}$

2) $a = bq + r$

Polynomials.

1) $\alpha + \beta = -\frac{b}{a}$

2) $\alpha\beta = \frac{c}{a}$

3) $P(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

Quadratic equations.

1) $Quadratic\ Equation : ax^2 + bx + c = 0$

2) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2) $Discriminant = \sqrt{b^2 - 4ac}$

Trigonometric ratios :

1) $\mathrm{Sin\theta} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

2) $\mathrm{Cos\theta} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$

3) $\mathrm{tan\theta} = \frac{\text{Opposite side}}{\text{Adjacent side}}$

Reciprocal Ratios :

1) $\mathrm{Sin\theta} = \frac{1}{\mathrm{Cosec\theta}}$ OR $\mathrm{Cosec\theta} = \frac{1}{\mathrm{Sin\theta}}$

2) $\mathrm{Cos\theta} = \frac{1}{\mathrm{Sec\theta}}$ OR $\mathrm{Sec\theta} = \frac{1}{\mathrm{Cos\theta}}$

3) $\mathrm{tan\theta} = \frac{1}{\mathrm{Cot\theta}}$ OR $\mathrm{Cot\theta} = \frac{1}{\mathrm{tan\theta}}$

4) $tan\theta = \frac{sin\theta}{cos\theta}$ $cot\theta = \frac{cos\theta}{sin\theta}$

Trigonometric Ratios of Standard angles

θ	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND

Trigonometric Identities :

1) $sin^2\theta + cos^2\theta = 1$

$sin^2\theta = 1 - cos^2\theta$

$cos^2\theta = 1 - sin^2\theta$

2) $1 + tan^2\theta = sec^2\theta$

$tan^2\theta = sec^2\theta - 1$

$sec^2\theta - tan^2\theta = 1$

3) $1 + cot^2\theta = cosec^2\theta$

$cot^2\theta = cosec^2\theta - 1$

$cot^2\theta - cosec^2\theta = 1$

Trigonometric ratios of complementary angles:

1) $\sin(90 - \theta) = cos\theta$

2) $\cos(90 - \theta) = sin\theta$

3) $\tan(90 - \theta) = cot\theta$

Statistics

1) $Mean = \frac{\sum f_i x_i}{\sum f_i}$

2) $Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

3) $Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

4) 3 Median = Mode + 2 Mean.

Probability

1) $P(E) = \frac{n(E)}{n(S)}$

2) $0 \leq P(E) \leq 1$

3) P(E) + P(not E) = 1

Surface Areas and Volumes.

1) $LSA\ of\ cube = 4a^2$
2) $TSA\ of\ cube = 6a^2$
3) $Volume\ of\ cube = a^3$
4) $LSA\ of\ cuboid = 2(bh + bl)$
5) $TSA\ of\ cuboid = 2(lb + bh + hl)$
6) $Volume\ of\ cuboid = l \times b \times h$

7) $CSA\ of\ cylinder = 2\pi rh$
8) $TSA\ of\ cylinder = 2\pi r(r + h)$
9) $Volume\ of\ cylinder = \pi r^2 h$

10) $CSA\ of\ cone = \pi rl$
11) $TSA\ of\ cone = \pi r(r + l)$
12) $Volume\ of\ cone = \frac{1}{3}\pi r^2 h$
13) $l^2 = r^2 + h^2$

14) $Surface\ area\ of\ a\ sphere = 4\pi r^2$
15) $Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$
16) $PSA\ of\ a\ hemisphere = \pi r^2$
17) $CSA\ of\ hemisphere = 2\pi r^2$
18) $TSA\ of\ a\ hemisphere = 3\pi r^2$
19) $Volume\ of\ a\ hemisphere = \frac{2}{3}\pi r^3$
20) $Volume\ of\ the\ frustum\ of\ cone$
$= \frac{1}{3}\pi({r_1}^2 + {r_2}^2 + r_1 r_2)h$
21) $CSA\ of\ frustum = \pi(r_1 + r_2)l$
22) $TSA\ of\ frustum\ of\ cone$
$= \pi(r_1 + r_2)l + \pi {r_1}^2 + \pi {r_2}^2$
23) $Slant\ height\ of\ frustum\ is\ given\ by$
$l = \sqrt{h^2 + (r_1 - r_2)^2}$