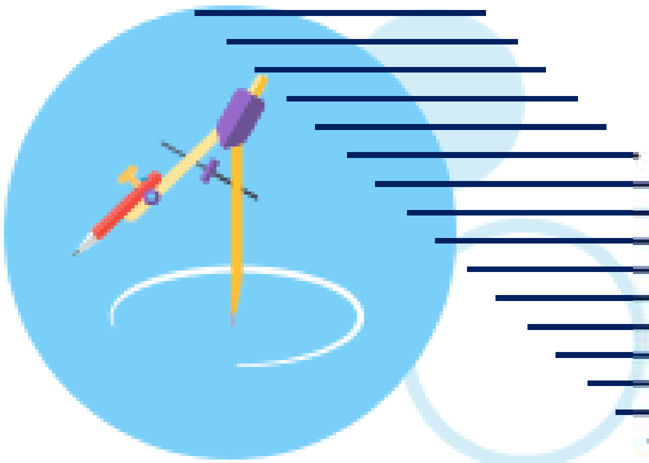
SSLC

Mathematics

Quick Glance



Prepared by YK

Arithmetic progression

Capter 1

An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

Let us denote the first term of an AP by a_1, second term by $a_2 \ldots n^{th}$ term by a_n and the common difference by d. Then the AP becomes $a_1, a_2, a_3, \ldots, a_n$.

So, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$, Then,

$$a, \; a + d, \; a + 2d, \; a + 3d, \ldots$$

This is called the general form of an AP.

Finite AP.:

In an AP there are only a finite number of terms. Such an AP is called a finite AP. Each of these Arithmetic Progressions (APs) has a last term.

Infinite AP.:

In an AP there are infinite number of terms. Such an AP is called a infinite AP. Each of these Arithmetic Progressions (APs) do not have last term.

n^{th} Term of an AP

The first term of an AP is a′ Common difference is d′ then the n^{th} term is $a_n = a + (n-1)d$

n^{th} term from the last n [l–last term , d – Common difference : $l - (n-1)d$

1.4 Sum of First n Terms of an AP

- ***First term - a Common difference - d***

$$S = \frac{n}{2}[2a + (n - 1)d]$$

- ***When the first and the last terms of an AP are given and the common difference is not given***

$$S = \frac{n}{2}[a + l]$$

Similar Figures

Two polygons of the same number of sides are similar, if

- *All the corresponding angles are equal and*
- *All the corresponding sides are in the same ratio (or proportion).*

Similarity of Triangles

Basic proportionality theorem [Thales theorem]

Theorem 2.1 *If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio*

Data: In ΔABC, the line drawn parallel to BC intersects AB and AC at D and E .

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: Join BE and CD. Draw DM⊥AC and EN⊥AB.

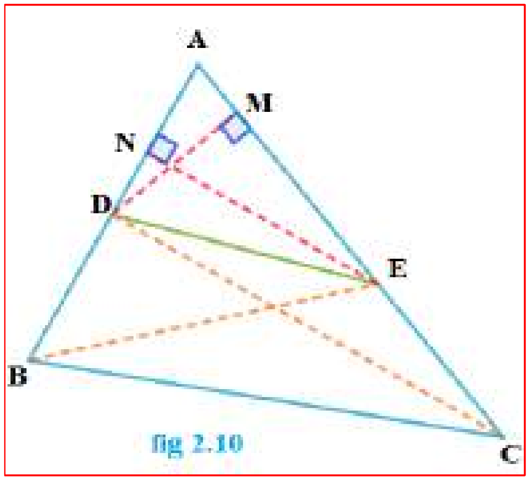


fig 2.10

Proof :

$$\frac{Area(\Delta ADE)}{Area(\Delta BDE)} = \frac{\frac{1}{2}\times AD \times EN}{\frac{1}{2}\times DB \times EN} = \frac{AD}{DB}$$ [∵Area of triangle = $\frac{1}{2}$ x Base x Height]

$$\frac{Area(\Delta ADE)}{Area\ (\Delta CED)} = \frac{\frac{1}{2}\times AE \times DM}{\frac{1}{2}\times EC \times DM} = \frac{AE}{EC}$$

ΔBDE and ΔDEC stand on the same base DE and in between BC∥DE

∴ Area (ΔBDE) = Area (ΔDEC) --- (3)

∴ $\frac{AD}{DB} = \frac{AE}{EC}$ [From (1), (2) and (3),]

Theorem 2.2 ***If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.***

It must be noted that as done in the case of congruency of two triangles, the similarity of two triangles should also be expressed symbolically, using correct correspondence of their vertices. For example, for the triangles ABC and DEF of Fig. 2.22, we cannot write ABC ~ EDF or ABC ~ FED. However, we can write BAC ~ EDF

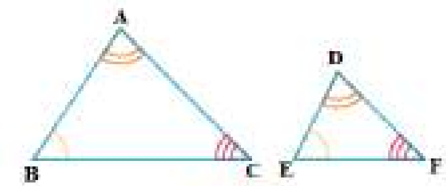


Fig 2.22

Theorem 2.3

AAA *If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.*

This criterion is referred to as the AAA (Angle–Angle–Angle) cr it er ion of similarity of two triangles.

Data: In ΔABC and ΔDEF, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

To prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (<1) and $\Delta ABC \sim \Delta DEF$

Construction: Cut $DP = AB$ from DE and $DQ = AC$ from DF and join PQ

Proof: In ΔABC and ΔDPQ,

$AB = DP$; $AC = DQ$ [Construction]

$\angle A = \angle D$ [data]

$\therefore \Delta ABC \cong \Delta DPQ$ [SAS Congruency rule]

$\Rightarrow$ **$BC = PQ$** --------- **(1)** and

$\angle B = \angle P$ [CPCT] But $\angle B = \angle E$ [Given]

$\therefore \angle P = \angle E$

$\therefore$ **$PQ \parallel EF$** [Since corresponding angles are equal]

$\therefore \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$ [by Corolary of BPT]

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ [By construction and (1)]

$\therefore \Delta ABC \sim \Delta DEF$

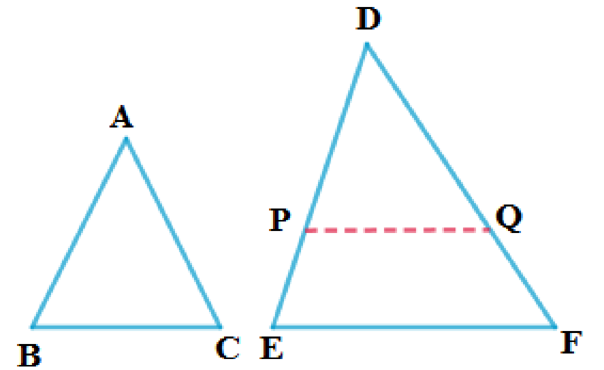


Fig 2.24

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This may be referred to as the AA similarity criterion for two triangles.

Theorem SSS

If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.

This criterion is referred to as the SSS (Side–Side–Side) similarity criterion for two triangles.

Data: In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ (<1) ---------(1)

To Prove: $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ And $\Delta ABC \cong \Delta DEF$

Construction: Cut DP = AB from DE and DQ = AC from DF.Join PQ

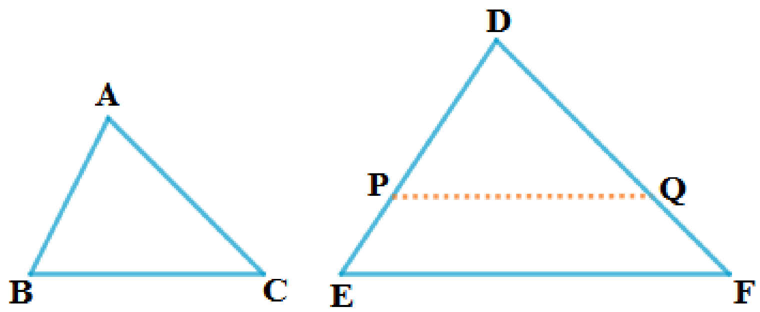


Fig 2.26

Proof : $\frac{AB}{DE} = \frac{AC}{DF}$ [Given]

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ [$\because$ DP = AB, DQ = AC]

$\therefore$ PQ∥EF [corollary of Converse of BPT in Δ DEF]

$\Rightarrow$ **∠P = ∠E ಮತ್ತು ∠Q = ∠F**

$\therefore$ $\Delta DPQ \sim \Delta DEF$ [AA Similarity criteria]

$\therefore \frac{DP}{DE} = \frac{PQ}{EF}$ [Corresponding sides of similar triangles]

$\Rightarrow \frac{AB}{DE} = \frac{PQ}{EF}$ ------(1) [AB = DP Construction] But, $\frac{AB}{DE} = \frac{BC}{EF}$ -------(2) [Given]

$\Rightarrow \frac{PQ}{EF} = \frac{BC}{EF}$ [$\because$ From (1) and (2)] $\Rightarrow$**BC = PQ**

In ΔABC and ΔDPQ ,

BC = PQ [Proved]; AB = DP [Construction]; AC = DQ [Construction]

$\therefore \Delta ABC \cong \Delta DPQ$ [SSS Congruency rule]

Hence, $\angle A = \angle D$, $\angle B = \angle P$ and $\angle C = \angle Q$ $\Rightarrow \angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ and $\Delta ABC \cong \Delta DEF$

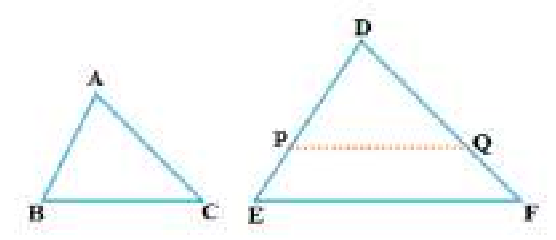
Theorem 2.5

Theorem 2.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar

Given: In ΔABC and ΔDEF, $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$ (< 1) ------------ (1)

To Prove: $\Delta ABC \cong \Delta DEF$

Construction: Cut DP = AB from DE and DQ = AC from DF.Join PQ



Proof: In ΔABC and ΔDPQ,

AB = PQ [By Construction]; AC = DF [By Construction]; $\angle A = \angle D$ [Given]

$\Delta ABC \cong \Delta DPQ$ [By SAS Congruency rule]----------(2)

From eqn (1) we get,

$\frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ *[AB = DP and AC = DQ]*

⇒ ***PQ***∥***EF*** *[By converse of corollary of BPT]*

$\Rightarrow \angle P = \angle E$, $\angle Q = \angle F$ *[Corresponding angles]*

∴ ΔDPQ ~ ΔDEF *[by AA similarity ctriteria] --------(3)*

⇒ ΔABC ≅ ΔDEF *[From equation (2) and (3)]*

Areas of Similar Triangles

Theorem 2.6

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given: ΔABC ~ ΔPQR

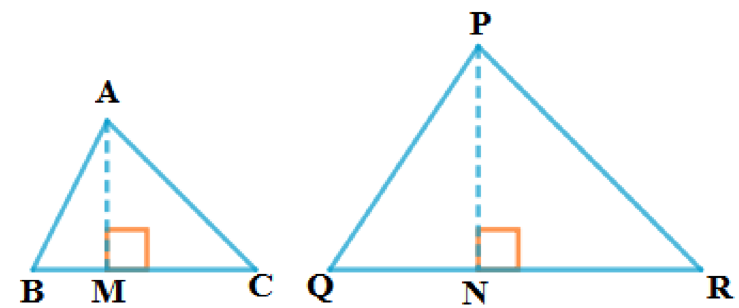


Fig 2.42

To Prove: $\frac{Area(ABC)}{Area(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{PR}\right)^2$

Construction: *Draw AM ⊥ BC and PN ⊥ QR*

Proof: $\frac{Area(ABC)}{Area(PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC \times AM}{QR \times PN}$ *- - -(1)* *[Area of triangle=$\frac{1}{2}$xbasexheight]*

In ΔABM and ΔPQN,

$\angle B = \angle Q$ *[Corresponding angles of the similar triangle]*

$\angle M = \angle N = 90^0$ *[Construction]*

∴ ΔABM ~ ΔPQN *[AA similarity criteria]*

$\Rightarrow \frac{AM}{PN} = \frac{AB}{PQ}$ - - - - - - (2) *But, ΔABC ~ ΔPQR [Given]*

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR}$ - - - - (3) $\Rightarrow \frac{AM}{PN} = \frac{BC}{QR}$ [From (2) and (3)]

$\therefore \frac{Area(ABC)}{Area(PQR)} = \frac{BC}{QR} \times \frac{BC}{QR}$ - -------- *[From (1) and (3)]* $\Rightarrow \frac{Area(ABC)}{Area(PQR)} = \left(\frac{BC}{QR}\right)^2$

$\frac{Area(ABC)}{Area(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{PR}\right)^2$ [From (3)]

Theorem 2.7

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Pythagoras Theorem

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Theorem 2.9

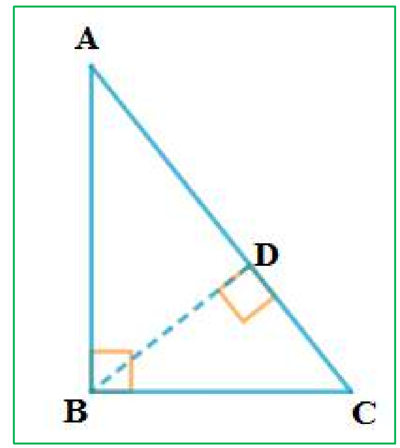
In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Theorem 2.8: [Pythagoras Theorem] In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: *In* ΔABC, $\angle B = 90^0$

To Prove: $AC^2 = AB^2 + BC^2$

Consruction: *Draw* $BD \perp AC$



Proof: *In* ΔADB *and* ΔABC

$\angle B = \angle D = 90^0$; $\angle A = \angle A$ *[Common angle]*

$\Delta ADB \sim \Delta ABC$ *[AA similarity criteria]*

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AD.AC = AB^2$ --------- (1)

In ΔBDC *and* ΔABC; $\angle B = \angle D = 90^0$

$\angle C = \angle C$ *[Common angle]*

$\Delta BDC \sim \Delta ABC$ *[AA similarity criteria]*

$\Rightarrow \frac{CD}{BC} = \frac{BC}{AC} \Rightarrow CD.AC = BC^2$ --------- (2)

$AD.AC + CD.AC = AB^2 + BC^2$ *[By adding (1) and (2)]*

$\Rightarrow AC\,(AD+CD) = AB^2 + BC^2$

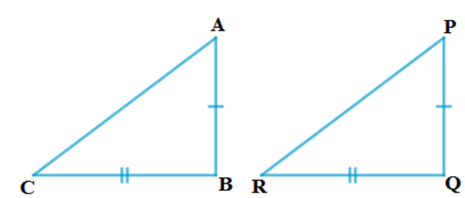
$\Rightarrow AC \times AC = AB^2 + BC^2 \Rightarrow AC2 = AB^2 + BC^2$

Theorem2.9: In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given: *In* ΔABC, $AC^2 = AB^2 + BC^2$

To prove: $\angle B = 90^0$

Construction: *Draw* ΔPQR *such that*



$\angle Q = 90^0$ *and* $PQ = AB$, $QR = BC$

Proof : *In* ΔPQR, $PR^2 = PQ^2 + QR^2$ *[by Pythogoras theorem]*

$PR^2 = AB^2 + BC^2$ *[Construction]* ----------- (1)

But, $AC^2 = AB^2 + BC^2$ [Given] -----------(2)

∴ AC = PR [from (1) and (2)] -----------(3)

AB = PQ [Construction]; BC = QR [Construction]

AC = PR [from (3)]

∴ ΔABC ≅ ΔPQR [SSS congruency rule] ∴ ∠B = ∠Q [By CPCT] But, ∠Q = 90^0 [Construction] ∴ **∠B = 90^0**

Chapter 3 Pair of Linear Equations intwo Variables

Linear equation with one variable: ***The algebraic equation of the type $ax + b = 0$ ($a \neq 0$ and b are real numbers, $x -$ variable is called linear equation of one variable. These type of equations having only one solution.***

Pair of Linear Equations in Two Variables

An equation which can be put in the form ax + by + c = 0, where a, b and c are real numbers, and a and b are not both zero, is called a linear equation in two variables x and y. A solution of such an equation is a pair of values, one for x and the other for y, which makes the two sides of the equation equal.

In fact, this is true for any linear equation, that is, each solution (x, y) of a linear equation in two variables, ax + by + c = 0, corresponds to a point on the line representing the equation, and vice versa.

2x + 3y=5; x - 2y - 3 = 0

These two linear equations are in the same two variables x and y. Equations like these are called a ***pair of linear equations in two variables.***

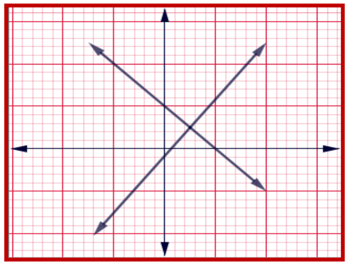
The general form for a pair of linear equations in two variables x and y is,

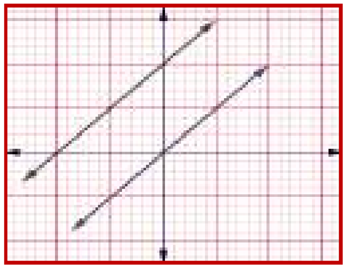
$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2x + c_2 = 0$

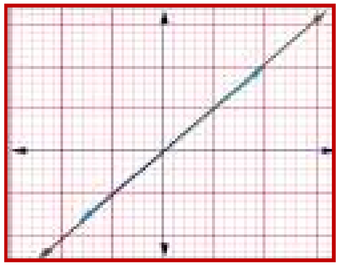
Here, $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers

Two lines in a plane, only one of the following three possibilities can happen:

(i) The two lines will intersect at one point. (ii) The two lines will not intersect, i.e., they are parallel. (iii) The two lines will be coincident.







Graphical Method of Solution of a Pair of Linear Equations

Consistent pair : *A pair of linear equations in two variables, which has a solution, is called a consistent pair of linear equations.*

Dependent pair : *A pair of linear equations which are equivalent has infinitely many distinct common solutions. Such a pair is called a dependent pair of linear equations in two variables.*

Inconsistent pair : *A pair of linear equations which has no solution, is called an inconsistent pair of linear equations.*

For the equations, $a_1x + b_1y + c_1 = 0$ ***and*** $a_2x + b_2y + c_2 = 0$

Comparing the ratios	Representing on graph	Algebraic solution	Consistency
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting each other	Unique solution	Consistant
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	coincident lines.	Infinite sulutions	dependent
$\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solutions	Inconsistent

Algebraic Methods of Solving a Pair of Linear Equations

In the previous section, we discussed how to solve a pair of linear equations graphically. The graphical method is not convenient in cases when the point representing the solution of the linear equations has non-integral coordinates

Substitution Method :

Step 1 : *Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.*

Step 2 : *Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved.*

Step 3 : *Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.*

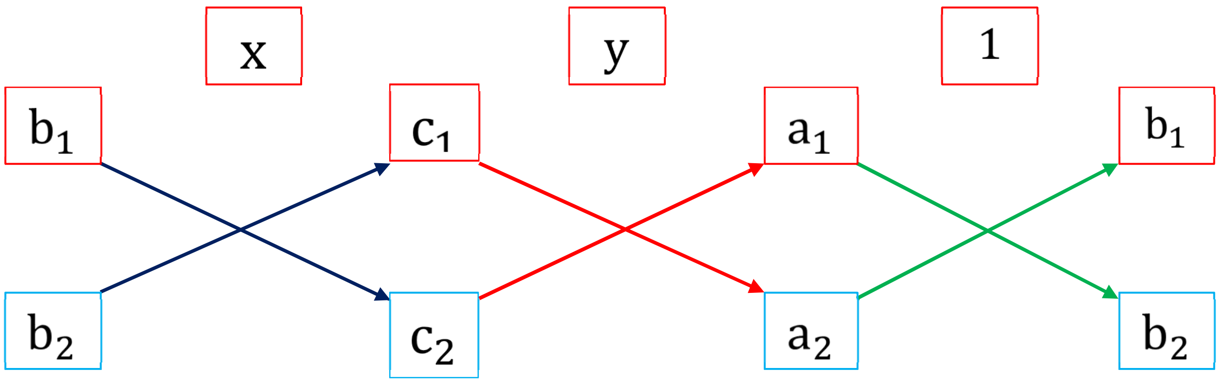
We have substituted the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why the method is known as the substitution method.

Cross - Multiplication Method

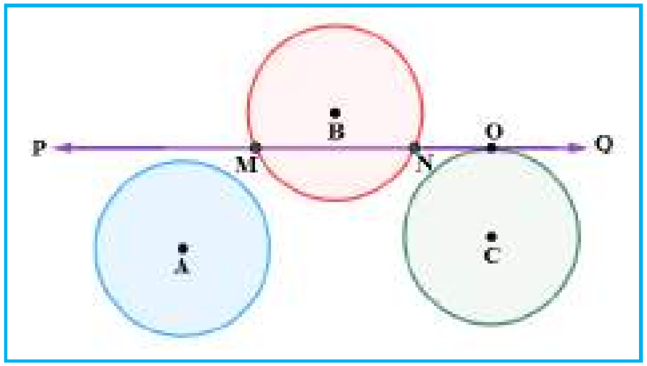
Equations are:

$$a_1x + b_1y + c_1 = 0; \quad a_2x + b_2y + c_2 = 0$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \qquad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad [a_1b_2 - a_2b_1 \neq 0] \qquad \frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$



Chapter 4 Circles



Non-intersecting Line: *The line PQ and the circle have no common point. In this case, PQ is called a non-intersecting line.PQ is non-intersecting line for the circle of center A*

Secant: *There are two common points M and N that the line PQ and the circle have. In this case, we call the line PQ a secant of the circle of center B*

Tangent: *There is only one point O which is common to the line PQ and the circle. In this case, the line is called a tangent to the circle of center C*

Tangent to a Circle

Tangent to a circle is a line that intersects the circle at only one point. There is only one tangent to a circle at a point. The common point of the tangent and the circle is called the point of contact.

Theorem 4.1 *The tangent at any point of a circle is perpendicular to the radius through the point of contact.*

Given: *A circle with centre O and tangent XY at a point P.*

To Prove: *OP⊥XY*

Consruction: *Take any point Q, other than P on the tangent XY and join OQ*

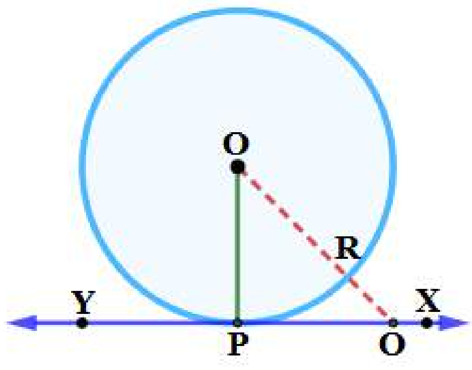
Proof: *Hence, Q is a point on the tangent XY, other than the point of contact P. So Q lies outside the circle..*

[∵There is only one point of contact to a tangent]

Let OQ intersect the circle at R
∴ OP = OR [∵ Radius of the same circle]
Now, OQ = OR + RQ

⇒OQ > OR ⇒OQ> OP [∵OP = OR]
Therefore, OP is the shortest distance to the tangent from the center O

∴ OP⊥XY [∵ Perpendicular distance is always the shortest distance]



Remarks :

1. By theorem above, we can also conclude that at any point on a circle there can be one and only one tangent.

2. The line containing the radius through the point of contact is also sometimes called the 'normal' to the circle at the point.

Number of Tangents from a Point on a Circle

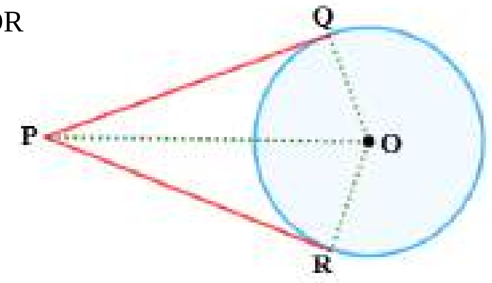
Case 1 : There is no tangent to a circle passing through a point lying inside the circle.

Case 2 : There is one and only one tangent to a circle passing through a point lying on the circle.

Case 3 : There are exactly two tangents to a circle through a point lying outside the circle.

Theorem 4.2 ***The lengths of tangents drawn from an external point to a circle are equal.***

Data: PQ and PR are the two tangents drawn from an external point P to a circle of center O. JoinOP, OQ, OR
T Prove: PQ = PR
Proof: In right angle triangle OQP and ORP,



OQ = OR [Radius of the same circle]
OP = OP [Common side]
∴ ΔOQ P ≅Δ ORP [RHS]
∴ PQ = PR [CPCT]

Chapter 5

Area Related to circles

Perimeter and Area of a Circle — A Review

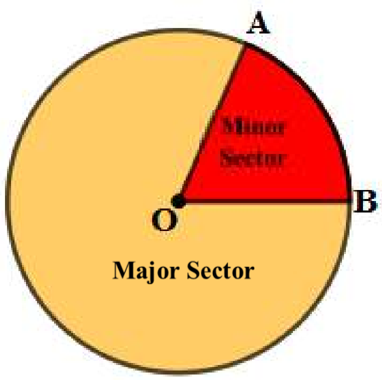
The distance covered by travelling once around a circle is its perimeter, usually called its circumference. You also know from your earlier classes, that circumference of a circle bears a constant ratio with its diameter. This constant ratio is denoted by the Greek letter π (read as 'pi'). In other words,

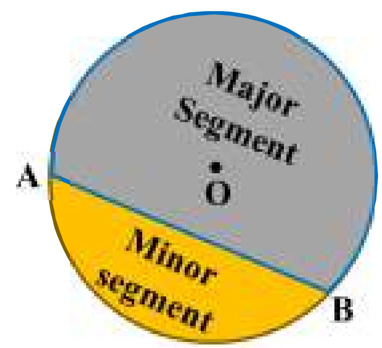
$\frac{\text{circumference}}{\text{diameter}}$ → Circumference $= 2\pi r$

Area of the triangle $= \pi r^2$

Areas of Sector and Segment of a Circle

The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called a sector of the circle and the portion (or part) of the circular region enclosed between a chord and the corresponding arc is called a segment of the circle.





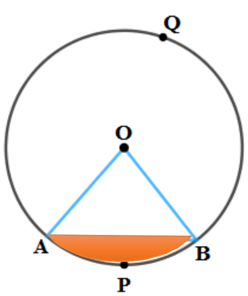
some relations (or formulae) to calculate their areas.

Let OAPB be a sector of a circle with centre O and radius r (see Fig. 5.6). Let the degree measure of ∠AOB be θ,
If the angle at the center is 360^0, then the area of the sector $= \pi r^2$

So, when the degree measure of the angle at the
Centre is 1, area of the sector $= \frac{\pi r^2}{360}$

Therefore, when the degree measure of the angle at the centre is θ,

Area of the sector $= \frac{\pi r^2}{360} \times \theta \Rightarrow \frac{\theta}{360} \times \pi r^2$



Area of the sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$; *Length of the arc of a sector of angle* $\theta = \frac{\theta}{360} \times 2\pi r$

Areaof SegmentAPB → **Area of OAPB Sector – Area of ΔOAB**

Area of OAQB Major sector → **πr^2 - Area of Minor sectorOAPB**

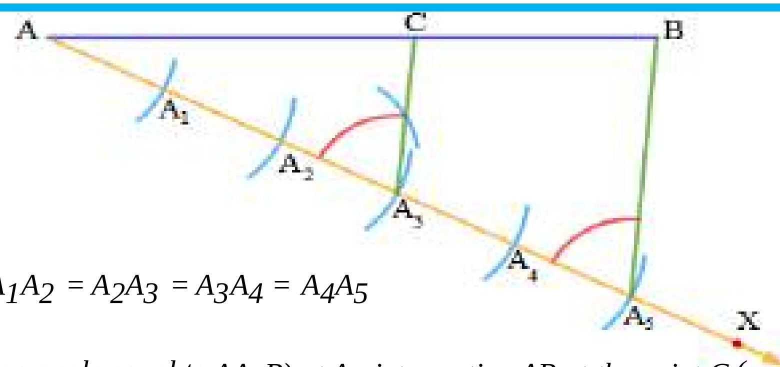
Area of AQB Major segment → **πr^2 – Area of APB Minor segment**

Chapter 6

Constructions

Division of a Line Segment

Construction 6.1: To divide a line segment in a given ratio.



Divede a line segment AB in the ratio m:n

Example: Devide the line segment AB in the ratio 3:2

Step-1: *Draw any ray AX, making an acute angle with AB*

(Can draw above or below the given line)

Step-2: *Locate 5 (= m + n) points* A_1, A_2, A_3, A_4 *and* A_5 *on AX so that* $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$

Step-3: *Join* BA_5

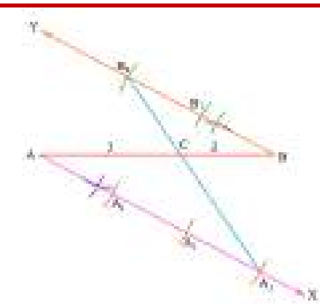
Step-4: *Through the point* A_3 *(m = 3), draw a line parallel to* A_5B *(by making an angle equal to* AA_5B*) at* A_3 *intersecting AB at the point C (see Fig.).*

Then, AC : CB = 3 : 2

Justification:

$A_3C \parallel A_5B \Rightarrow \frac{AA_3}{A_3A_5} = \frac{AC}{CB}$ *[Basic proportionality theorem]* $\Rightarrow \frac{AA_3}{A_3A_5} = \frac{3}{5-3} = \frac{3}{2} \Rightarrow 3{:}2$ **Now AC : CB = 3 : 2**

Alternate Method:



Step 1: *Draw any ray AX making an acute angle with AB .*

Step 2: *Draw a ray BY parallel to AX by making* $\angle ABY$ *equal to* $\angle BAX$

Step 3: *Locate the points* A_1, A_2, A_3 *(m = 3) on AX and* B_1, B_2 *(n = 2) on BY such that* $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$

Step 4: *Join* A_3B_2*. Let it intersect AB at a point C*

Justification:

In $\Delta\ AA_3C$ *and* $\Delta\ BB_2C$ $\angle\ ACA_3 = BCB_2$ *(Vertically opposite angles)*

$\angle\ CAA_3 = CBB_2$ *(Alternate angles)*

$\Delta\ AA_3C \sim \Delta\ BB_2C$ *(AA similarity criteria)*

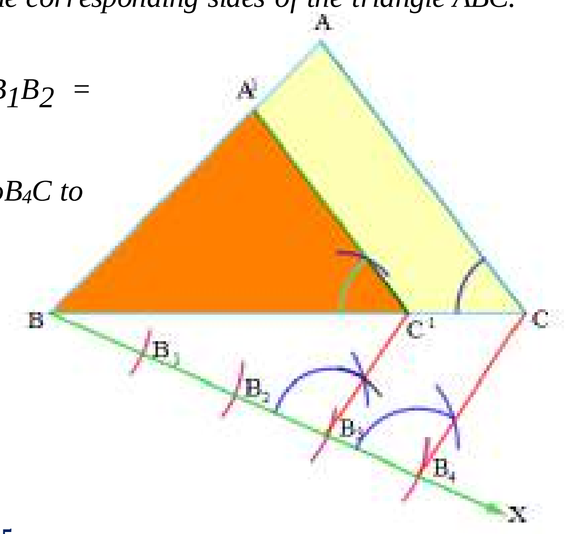
$\Rightarrow \frac{AA_3}{BB_2} = \frac{AC}{BC}$ *[BPT]* $\Rightarrow \frac{AA_3}{BB_2} = \frac{3}{2} \Rightarrow \frac{AC}{BC} = \frac{3}{2} \Rightarrow AC : BC = 3:2$

Construction 6.2:

To construct a triangle similar to a given triangle as per given scale factor.

Example1:Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of the corresponding side of the triangle ABC [i,e. of scale factor $\frac{3}{4}$]

Solution: *Given a triangle ABC, we are required to construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.*



Step-1: *Draw any ray BX making an acute angle with BC on the side opposite to the vertex A*

Step-2: *Locate 4 (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.*

Step-3: *Join B_4C and draw a line through B_3 the 3^{rd} point, (3 being smaller of 3 and 4in $\frac{3}{4}$)parallel toB_4C to intersect BC at C^1*

Step-3: *Draw a line through C^1 parallel to the line CA to intersect* BA at A^1

Justification:

$\frac{BC^1}{C^1C} = \frac{3}{1}$ $\therefore \frac{BC}{BC^1} = \frac{3+1}{3} = \frac{4}{3}$ $\Rightarrow \frac{BC^1}{BC} = \frac{3}{4}$

$C^1A^1 \parallel CA$ $\therefore \Delta\ A^1BC^1 \sim \Delta\ ABC$ $\Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{3}{4}$

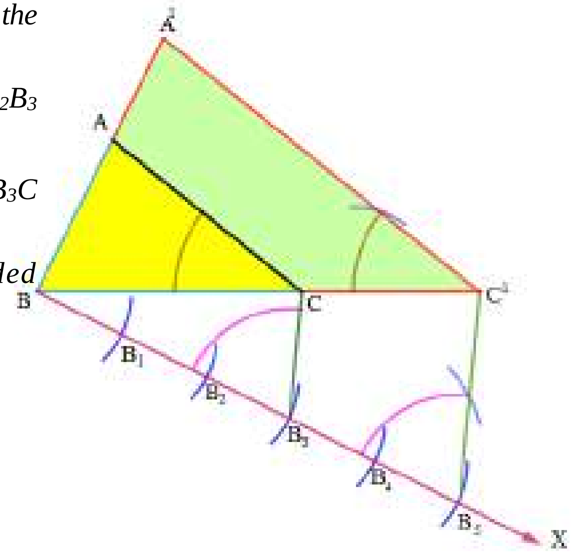
Example 2 : Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{5}{3}$ of the corresponding side of the triangle ABC [i,e. of scale factor $\frac{5}{3}$]

Step1: *Construct any ΔABC. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.*

Step 2: *Locate 5 points (the greater of 5 and 3 in* $\frac{5}{3}$*) B_1, B_2, B_3, B_4 and B_5 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$*

Step 3: *Join B_3(the 3rd point, 3 being smaller of 3 and 5 in* $\frac{5}{3}$ *) to C and draw a through B_5 parallel to B_3C intersect BC at C^1*

Step 4: *Draw a line through C^1 parallel to the line CA to intersect BA at A^1 [Note: Extended BA]*



Justification:

$\Delta\, ABC \sim \Delta\, A'BC \;\Rightarrow \frac{AB}{A^1B} = \frac{AC}{A^1C^1} = \frac{BC}{BC^1}$

But, $\frac{BC}{BC^1} = \frac{BB_3}{BB_5} = \frac{3}{5} \quad \therefore \frac{BC^1}{BC} = \frac{5}{3} \quad \Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{5}{3}$

Construction of Tangents to a Circle

To construct the tangents to a circle from a point outside it

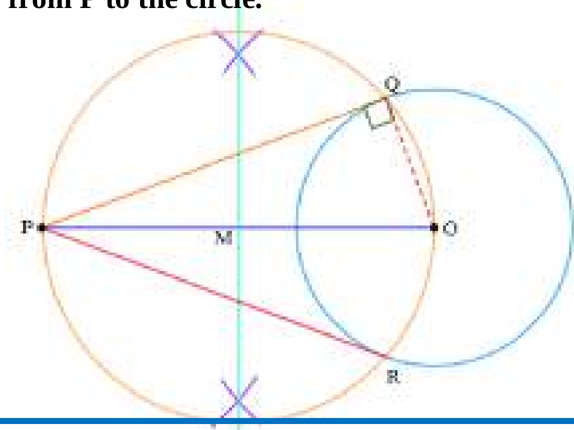
We are given a circle with centre O and a point P outside it. We have to construct the two tangents from P to the circle.

Step 1: *Join PO and bisect it. Let M be the mid- point of PO*

Step 2: *Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at the points Q and R.*

Step 3: *Join PQ and PR*

Then PQ and PR are the required two tangents



Justification:

Join OQ, ∠PQO is an angle in semi circle

∴ ∠ PQO = 90^0 ⇒ PQ ⊥OQ, OQ is the radius of given circle. Therefore PQ is the tangent to the circle. Similarlly PR also the tangent to the circle.

Chapter 7

Coordinate Geometry

Coordinate axes:

A set of a pair of perpendicular axes $X^|OX$ and $YOY^|$

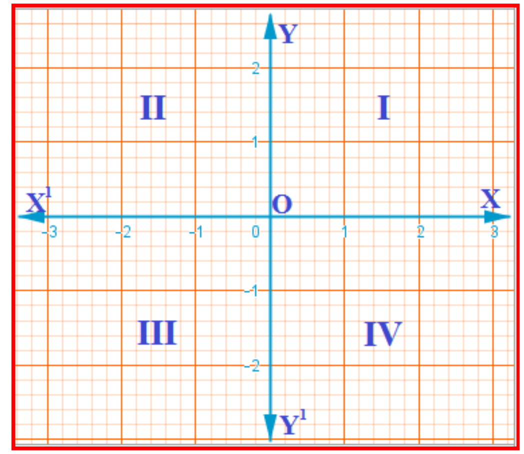
Horizontal line

X^1OX | X - Axis

Vertical line

YOY^1 | Y - Axis

The intersection point of X and Y axes is called the Orgin 'O'



The distance of a point from the y-axis is called its x-coordinate, or abscissa. The distance of a point from the x-axis is called its y-coordinate, or ordinate. The coordinates of a point on the x-axis are of the form (x, 0), and of a point on the y-axis are of the form (0, y).

The Coordinate axes divides the plane in to four parts. They are called quadrants.

The coordinaes of the orgin is $(\mathbf{0}, \mathbf{0})$

Distance Formula

The distance between two points on X-axis or on the straight line paralle to X-axis is

$$\textbf{Distance} = x_2 - x_1$$

The distance between two points on Y-axis or on the straight line paralle to Y-axis is

$$\textbf{Distance} = y_2 - y_1$$

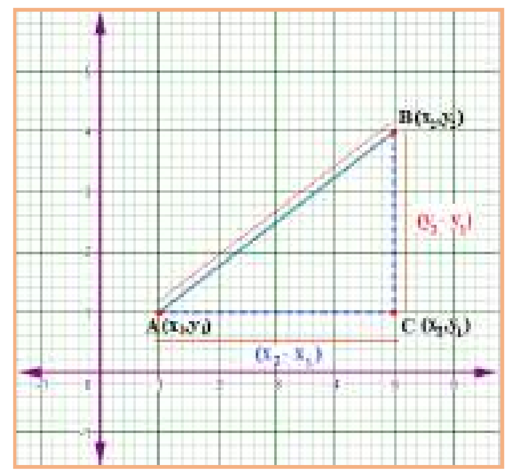
$AB^2 = AC^2 + BC^2$

The distance between two points which are neither on X or Y axis nor on the line paralle to X orY axis

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between the point P(x,y) and the orgin

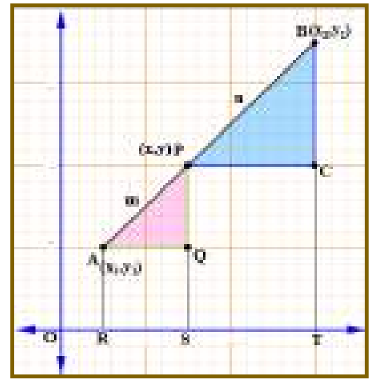
$$d = \sqrt{x^2 + y^2}$$



Section Formula

The coordinates of the point P(x, y) which divides the line segment joining points $\boldsymbol{A(x_1, y_1)}$ *and* $\boldsymbol{B(x_2,\ y_2)}$*, internally, in the ratio* $m_1 : m_2$ *are*

$$P(x, y) = \left(\frac{m_1 x_2 + m_2\ x_1}{m_1 + m_2}\ ,\ \frac{m_1 y_2 + m_2\ y_1}{m_1 + m_2}\right)$$



The mid-point of a line segment divides the line segment in the ratio 1: 1. Then the coordinates of the midpoint of the line segment,

$$P(x, y) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right)$$

Area of a Triangle

Area of triangle $= \frac{1}{2} \times$ *base* $\times$ *height*

By Heron's Formula Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ *, Here,* $s = \frac{a+b+c}{2}$

a, b and c are the sides of the triangle.

We could find the lengths of the three sides of the triangle using distance formula. But this could be tedious, particularly if the lengths of the sides are irrational number. Then we can use the following formula to find the area of the triangle.

Area of the triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Chapter 8 Real Numbers

Euclid's division algorithm, as the name suggests, has to do with divisibility of integers. Stated simply, it says any positive integer a can be divided by another positive integer b in such a way that it leaves a remainder r that is smaller than b.

Euclid's Division Lemma

Theorem 8.1

(Euclid's Division Lemma): Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, 0 r < b.

A lemma is a proven statement used for proving another statement

The Fundamental Theorem of Arithmetic

Theorem 8.2 (Fundamental Theorem of Arithmetic) : Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime the prime factors occur

The Fundamental Theorem of Arithmetic says that every composite number can be factorised as a product of primes. Actually it says more. It says that given any composite number it can be factorised as a product of prime numbers in a'unique' way, except for the order in which the primes occur. That is, given any composite number there is one and only one way to write it as a product of primes, as long as we are not particular about the order in which the primes occur. So, for example, we regard 2 × 3 × 5 × 7 as the same as 3 × 5 × 7 × 2, or any other possible order in which these primes are written.

Any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

We can use this result to find the LCM of two positive integers, if we have already found the HCF of the two positive integers.

Revisiting Irrational Numbers

A number which can not be expressed in the form of $\frac{p}{q}$ *is called irrational number.Here,* $p, q \epsilon Z, q \neq 0$

Theorem 8.3: Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Theorm 8.4: $\sqrt{2}$ is irrational.

Proof: Let us assume, to the contrary, that $\sqrt{2}$ *is rational.*

$\Rightarrow \sqrt{2} = \frac{p}{q}$ [$p,q \in Z, q \neq 0$ and $(p,q)=1$]

So, there is no other common factors for p and q other than 1

Now, $\sqrt{2} = \frac{p}{q} \Rightarrow \sqrt{2}q = p$ *Squaring on both sides we get,*

$\left(\sqrt{2}q\right)^2 = p^2 \quad \Rightarrow 2q^2 = p^2 \qquad (1)$

$\Rightarrow$ 2 divides $p^2 \Rightarrow$ 2, divides p . [By theorem]

$\therefore$ Let $p = 2m$,

(1) $\Rightarrow 2q^2 = (2m)^2 \Rightarrow q^2 = 2m^2$

$\Rightarrow$2,divides $q^2 \Rightarrow$ 2, divides q [By theorem]

$\therefore$ 2 is the common factor for both p and q

This contradicts that there is no common factor of p and q .

Therefore our assumption is wrong. So, $\sqrt{2}$ *is a an irrational number.*

- ***The sum or difference of a rational and an irrational number is irrational and***
- ***The product and quotient of a non-zero rational and irrational number is irrational.***

Revisiting Rational Numbers and Their Decimal Expansion:

Theorem 8.5: *Let x be a rational number whose decimal expansion terminates.Then x can be expressed in the form $\frac{p}{q}$ where p and q are coprime, and the prime factorisation of q is of the form $2^n.5^m$, where n, m are non-negative integers.*

Theorem 8.6 : *Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form 2n5m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.*

Theorem 8.7 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

Chapter 9 Polynomials

Degree of the polynomial:

p(x) is a polynomial in x, the highest power of x in p(x) is called the degree of the polynomial p(x).

*A polynomial of degree 1 is called a **linear polynomial.***

*A polynomial of degree 2 is called a **quadratic polynomial.***

quadratic polynomial in x is of the form $ax^2 + bx + c$, where a, b, c are real numbers $a \neq 0$.

is a polynomial in the variable x of degree 3

*A polynomial of degree 3 is called a **cubic polynomial**. General form of a cubic polynomial is*

$ax^3 + bx^2 + cx + d$

Where a, b, c, d are real numbers and $a \neq 0$

If k is the zero of the polynomial p(x) = ax + b then p(k) = ak + b = 0 ⇒ $k = -\frac{b}{a}$

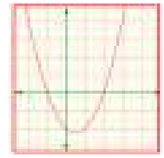
The zero of the lenear equation ax + b is $-\frac{b}{a}$

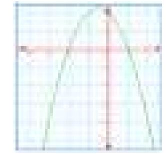
Geometrical Meaning of the Zeroes of a Polynomial

(i) Linear Polynomial

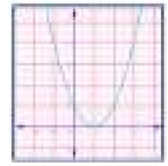
(i) Quadratic Polynomials:

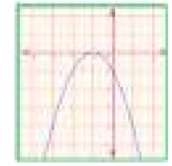
Case (i) : *Here, the graph cuts x-axis at two distinct points A and A^1. The x-coordinates of A and A^1 are the two zeroes of the quadratic polynomial $x^2 + bx + c$*



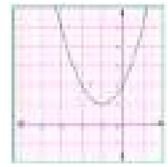


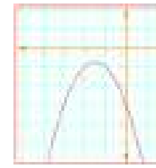
Case (ii) : *Here, the graph cuts the x - axis at exactly one point, i.e., at two coincident points. So, the two points A and A^1 of Case (i) coincide here to become one point A . The x –coordinate of A is the only zero for the quadratic polynomial $ax^2 + bx + c$*





Case (iii) : *Here, the graph is either completely above the x-axis or completely below the x-axis. So, it does not cut the x-axis at any point So, the quadratic polynomial $ax^2 + bx + c$ has no zero*





So, you can see geometrically that a quadratic polynomial can have either two distinct zeroes or two equal zeroes (i.e., one zero), or no zero. This also means that a polynomial of degree 2 has atmost two zeroes.

Cubic Polynomials:

Relationship between Zeroes and Coefficients of a Polynomial

α and β are the zeros of the polynomial $p(x) = ax^2 + bx + c, \quad a \neq 0$
(x -) and (x - β) are the factors of p(x).

Sum of Zeros $\boldsymbol{\alpha + \beta = \frac{-b}{a}}$ *Product of Zeros* $\boldsymbol{\alpha\beta = \frac{c}{a}}$

The relation between the zeros and the coefficients of Cubic polynomials:

If α, β, γ are the zeros of the cubic polynomia $ax^3 + bx^2 + cx + d$ then

$$\boldsymbol{\alpha + \beta + \gamma = \frac{-b}{a}; \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}; \quad \alpha\beta\gamma = \frac{-d}{a}}$$

Division Algorithm for Polynomials:

⇒ Dividend= Divisor x Quotient + Remainder

If p(x) and g(x) are any two polynomials and $g(x) \neq 0$ then,

p(x) = g(x) . q(x) + r(x) q(x) - Quotient and r(x) – remainder. Here, r(x) = 0 or the degree of r(x) < the degree of g(x)

This is known as The Division Algorithm for polynomials

When we equate this polynomial to zero, we get a quadratic equation.

Any equation of the form p(x) = 0, where p(x) is a polynomial of degree 2, is a quadratic equation.

Standard form of quadratic equations:

$\boldsymbol{ax^2 + bx + c = 0}$, *Where* $a \neq 0$

The features of quadratic equations:

- *The quadratic equations has one variable*
- *The hieghest power of the variable is 2*
- *Standard form of quadratic equation:* $ax^2 + bx + c = 0$,

Adfected quadratic equations : *In a quadratic equation* $ax^2 + bx + c = 0$, $a \neq 0$, $b \neq 0$ *then it is called adfected quadratic equations.*

Then, $x^2 - 3x - 5 = 0$, $x^2 + 5x + 6 = 0$, $x + \frac{1}{x} = 5$, $(2x - 5)^2 = 81$

Pure Quadratic equations : *The quadrtic equations where* $a \neq 0$, *b = 0 is called pure quadratic equations.*

The standard form of pure quadratic equation: $ax^2 + c = 0$ *[*$a \neq 0$*]*

Solution of a Quadratic Equation by Completing the Square

Nature of Roots

The value of $b^2 - 4ac$ *decides the roots of quadratic equation* $ax^2 + bx + c = 0$ *has real or not, therefore*

$b^2 - 4ac$ is called the discriminant of this quadratic equation.and denoted by Δ [delta]

So, the quadratic equation $ax^2 + bx + c = 0$ *has*

Discriminant	***Nature of the roots***
$\Delta = 0$	***Two equal real roots***
$\Delta > 0$	***Two distinct real roots***
$\Delta < 0$	***No real roots***

Chapter 11

ITRODUCTION TO TRIGONOMETRY

Trigonometry is the study of relationships between the sides and angles of a triangle.

11.2 Trigonometric Ratios:

There are six trigonometric ratios:

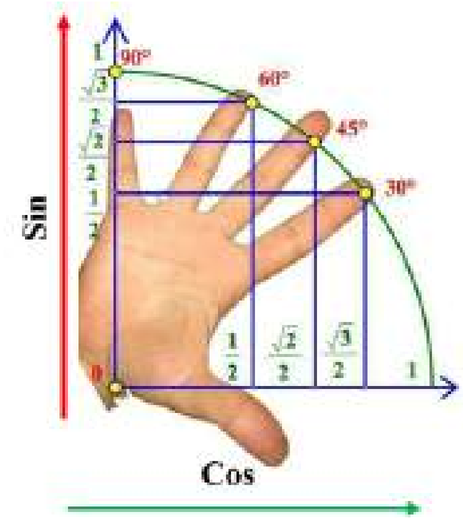
Trigonometric ratios		Triangle 1	Triangle 2
SinA	$\frac{Opposite}{Hypotenuse}$	$\frac{BC}{AC}$	$\frac{AB}{AC}$
CosA	$\frac{Adjecent}{Hypotenuse}$	$\frac{AB}{AC}$	$\frac{BC}{AB}$
Tan A	$\frac{Opposite}{Adjecent}$	$\frac{BC}{AB}$	$\frac{AB}{BC}$
CosecA	$\frac{Hypotenuse}{Opposite}$	$\frac{AC}{BC}$	$\frac{AC}{AB}$
SecA	$\frac{Hypotenuse}{Adjecent}$	$\frac{AC}{AB}$	$\frac{AC}{BC}$
CotA	$\frac{Adjecent}{Opposite}$	$\frac{AB}{BC}$	$\frac{BC}{AB}$

Inverse of trigonometric values		
$\frac{1}{SinA}$	$\frac{Hypotenuse}{Opposite}$	**CosecA**
$\frac{1}{CosA}$	$\frac{Hypotenuse}{Adjecent}$	**SecA**
$\frac{1}{Tan\ A}$	$\frac{Adjecent}{Opposite}$	**CotA**
$\frac{1}{CosecA}$	$\frac{Opposite}{Hypotenuse}$	**SinA**
$\frac{1}{SecA}$	$\frac{Adjecent}{Hypotenuse}$	**SecA**
$\frac{1}{CotA}$	$\frac{Opposite}{Adjecent}$	**CotA**

Trigonometric Ratios of Some Specific Angles:

∠A	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
osec	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
Cot	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

For curiosity



11.5 Trigonometric Identities

You may recall that an equation is called an identity when it is true for all values of the variables involved. Similarly, an e

ratios of an angle is called a trigonometric identity, if it is true for all values of the angle(s) involved.

$\boldsymbol{Sin^2A + cos^2A = 1}$

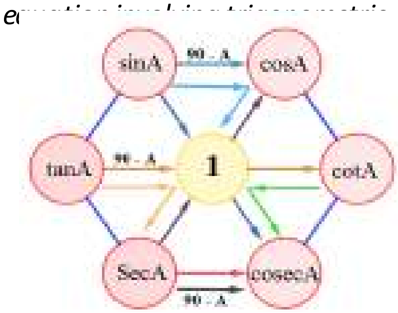
$\boldsymbol{tan^2 + 1 = sec^2A}$

$\boldsymbol{1 + cot^2A = cosec^2A}$

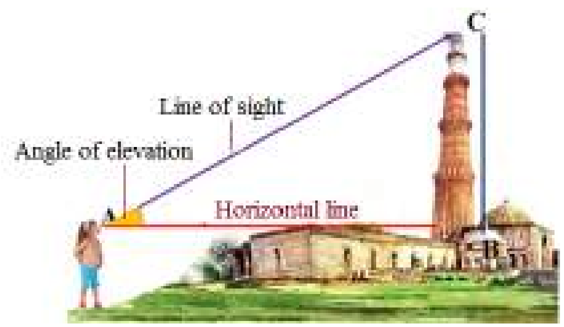
Note: $\frac{sinA}{cosA} = tanA$

$\frac{cosA}{sinA} = cotA$

For curiosity



12 *Some Applications of Trigonometry*



Trigonometry is one of the most ancient subjects studied by scholars all over the world. As we have said in Chapter 11, trigonometry was invented because its need arose in astronomy. Since then the astronomers have used it, for instance, to calculate distances from the Earth to the planets and stars. Trigonometry is also used in geography and in navigation. The knowledge of trigonometry is used to construct maps, determine the position of an island in relation to the longitudes and latitudes. Surveyors have used trigonometry for centuries. One such large surveying project of the nineteenth century was the 'Great Trigonometric Survey' of British India for which the two largest-ever theodolites were built. During the survey in 1852, the highest mountain in the

world was discovered. From a distance of over160 km, the peak was observed from six different stations. In 1856, this peak was named after Sir George Everest, who had commissioned and first used the giant theodolites (see the figure alongside). The theodolites are now on display in the Museum of the Survey of India in Dehradun.

12.2 Height and distance:

Thus, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer. The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

Thus, the angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed

Chapter13

Statistics

Mean of Grouped data :

Direct Method to find average: **Average:** $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ **[i = 1 to n]**

Assumed Mean Method: *Average* $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$

Step Deviation Method: $d_i = x_i - a$ **;** *Average* $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

Note: *If all di have common multiple then step deviation method is the best method*

We get the same average in all three methods.
Assumed Mean and step deviation methods are the simplified form of Direct Method.

Mode of Grouped Data

A mode is that value among the observations which occurs most often, that is, the value of the observation having the maximum frequency

Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

L = lower limit of the modal class
h = size of the class interval (assuming all class sizes to be equal),
f_1 ***= frequency of the modal class,***
f_0 ***= frequency of the class preceding the modal class,***
f_2 ***= frequency of the class succeeding the modal class***

Median of Grouped Data

the median is a measure of central tendency which gives the value of the middle-most observation in the data. Recall that for finding the median of ungrouped data, we first arrange the data values of the observations in ascending order, then, if n is odd, then the meadian is $\left(\frac{n+1}{2}\right)$*th observation and if n is an even, then the dedian is the average of* $\left(\frac{n}{2}\right)$ *and* $\left(\frac{n}{2}+1\right)$ *th observation.*

After finding the median class, we use the following formula for calculating the median.

Median of Grouped Data

$$Median = l + \left[\frac{\frac{n}{2} - cf}{f}\right] x\ h$$

l = lower limit of median class,
n = number of observations
cf = cumulative frequency of class preceding the median class,.
f = frequency of median class
h = class size (assuming class size to be equal).

Graphical Representation

Chapter 14

Probabilty

Probability — A Theoretical Approach

Suppose a coin is tossed at random
the coin can only land in one of two possible ways — either head up or tail up.
suppose we throw a die once. For us, a die will always mean a fair die. They are 1, 2, 3, 4, 5, 6.
Each number has the same possibility of showing up.

The experimental or empirical probability P(E) of an event E as

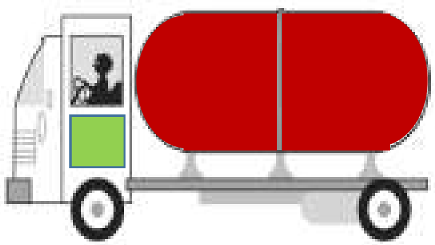
$$P(E) = \frac{Number\ of\ trials\ in\ which\ the\ event\ happened}{Total\ number\ of\ trails}$$

The theoretical probability (also called classical probability) of an event E, written as P(E), is defined as

$$P(E) = \frac{No\ of\ outcomes\ favarable\ to\ 'E}{No.of\ all\ possible\ outcomes\ of\ the\ experiment}$$

P(A) = 1 – P($\bar{A}$) : where A is an event and $\bar{A}$ is complement of an event A
*That is, the probability of an event which is impossible to occur is 0. Such an event is called an **impossible** event*
*So, the probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a certain event.*

Surface Area and Volumes



Surface Area of a Combination of Solids

To find the surface area or the volume of a container or test tube we have to break it up two or more known solids. For example,

Area of the container

= Area of the hemisphere + Area of the cylinder + Area of the hemisphere

Conversion of Solid from One Shape to Another

We can convert one shape to another. When we convert the shape, the volume of the new shape will be the same as the earliar shape.

60 = 100 min

Frustum of a Cone

Given a cone, when we slice (or cut) through it with a plane parallel to its base (see Fig. 15.20) and remove the cone that is formed on one side of that plane, the part that is now left over on the other side of the plane is called a frustum of the cone.

Example 12 : The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm (see Fig. 15.21). Find its volume, the curved sur face area and the total suface area (take $\pi = \frac{22}{7}$)

Volume of frustum of cone $= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$

CSA of frustum of cone $= \pi(r_1 + r_2)l$ [$l = \sqrt{h^2 + (r_1 - r_2)^2}$

TSA of frustum: $= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$