"ಸಂಜೀವಿನಿ"

ಗಣಿತ

MATHEMATICS

ನೋಡಲ್ ಅಧಿಕಾರಿಗಳು:

ಶ್ರೀಮತಿ ಶುಭ ಎಂ
ನೋಡಲ್ ಅಧಿಕಾರಿಗಳು,
ಕರ್ನಾಟಕ ಪಬ್ಲಿಕ ಶಾಲೆ ಮಂಚೆಗೌಡನ ಕೊಪ್ಪಲು,
ಮೈಸೂರು ತಾಲ್ಲೂಕು, ಮೈಸೂರು.

ಸಂಪನ್ಮೂಲ ಶಿಕ್ಷಕರ ತಂಡ

ಶ್ರೀ ವಿಜಯಸಿಂಹನ್ ಎಂ ಎನ್
ಗಣಿತ ಶಿಕ್ಷಕರು
ಸರ್ಕಾರಿ ಪ್ರೌಢಶಾಲೆ ಬೀರಿಹುಂಡಿ
ಮೈಸೂರು ತಾಲ್ಲೂಕು ಮತ್ತು ಜಿಲ್ಲೆ

ಶ್ರೀ ಅರುಣಕುಮಾರ ಸಿ ಬಿ
ಗಣಿತ ಶಿಕ್ಷಕರು
ಸರ್ಕಾರಿ ಪ್ರೌಢಶಾಲೆ ಕಲ್ಲೂರು ನಾಗನಹಳ್ಳಿ
ಮೈಸೂರು ತಾಲ್ಲೂಕು ಮತ್ತು ಜಿಲ್ಲೆ.

ಅನುವಾದಕ ಶಿಕ್ಷಕರ ತಂಡ

ಶ್ರೀಮತಿ ಪುಷ್ಪ ಜೆ
ಗಣಿತ ಶಿಕ್ಷಕರು
ಕರ್ನಾಟಕ ಪಬ್ಲಿಕ ಶಾಲೆ ಮಂಚೆಗೌಡನ ಕೊಪ್ಪಲು
ಮೈಸೂರು ತಾಲ್ಲೂಕು ಮತ್ತು ಜಿಲ್ಲೆ.

ಶ್ರೀಮತಿ ಅಕ್ಕಮಹಾದೇವಿ ಮುಕ್ಕನಗೌಡರ
ಗಣಿತ ಶಿಕ್ಷಕರು
ಕರ್ನಾಟಕ ಪಬ್ಲಿಕ ಶಾಲೆ ಮಂಚೆಗೌಡನ ಕೊಪ್ಪಲು
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ಶ್ರೀಮತಿ ಅರ್ಚನ
ಗಣಿತ ಶಿಕ್ಷಕರು
ರೋಟರಿ ಮಿಡ್ ಟೌನ್ ಹೂಟಗಳ್ಳಿ
ಮೈಸೂರು ತಾಲೂಕು ಮತ್ತು ಜಿಲ್ಲೆ.

ಶ್ರೀಮತಿ ಉಷಾ ಎನ್
ಗಣಿತ ಶಿಕ್ಷಕರು
ವಿಜಯ ಪ್ರೌಢಶಾಲೆ ಹಿನಕಲ್,
ಮೈಸೂರು ತಾಲೂಕು ಮತ್ತು ಜಿಲ್ಲೆ.

1.ARITHMETIC PROGRESSION

General form of an A.P : $a, (a+d), (a+2d) \ldots\ldots$
The first term is $'a'$,common difference is $'d'$ then n[th] term is $a_n = a + (n-1)d$
The first term is $'a'$,common difference is $'d'$ sum to n terms is $Sn = \frac{n}{2}\ [\ 2a + (n-1)d\ ]$
The first term is $'a'$, and n[th] term is $'a_n$ then sum to n terms is $Sn = \frac{n}{2}\ (\ a + a_n\ )$
Sum to first $'n'$ terms of positive integers (natural numbers) $S_n = \frac{n(n+1)}{2}$
If a,A,b are in Arithmetic progression then $A = \frac{a+b}{2}$
$s_1 = a_1$ $s_2 = a_1 + a_2$ $s_3 = a_1 + a_2 + a_3$

TO FIND THE n^{th} TERM OF AN A.P

1) Find the 20^{th} term of 3, 8, 13, 18. . . .

$n = 20\ a = 3,\ d = 8-3 = 5,\quad a_{20}=?$

$a_n = a + (n-1)d$

$a_{20} = 3 + (20-1)5$

$a_{20} = 3 + 19 \times 5$

$a_{20} = 98$

For Practice

1) Find the 25^{th} term of 6, 10, 14,............. using formula.

2) Find the 20^{th} term of 4,7, 10.............. using formula

3) Find the 12^{th} term of 7, 13, 19.........using formula

4) Find the 10^{th} term of 11, 8, 5, 2 ....using formula.

5) Find the 10^{th} term of 15, $\frac{31}{2}$, 16............... using formula.

6) Find the 50^{th} term of 7, 12, 17, ... using formula.

7) Find the 30^{th} term of 5, 8, 11 .... using formula.

8) Find the 12^{th} term of 2, 5, 8.................... using formula.

To find the sum to ' n' terms of an Arithmetic progression

1) F ind the sum to first 10 terms of 2, 7, 12, ... using formula

$a = 2,\ d = 7-2 = 5,\ S_{10} = ?\quad n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(2) + (10-1)5]$

$S_{10} = 5[4 + 9(5)]$

$S_{10} = 5[4 + 45]$

$S_{10} = 5 \times 49$

$= 245$

For Practice

1) 2 + 7 + 12 + ……………. Find S_{20} using suitable formula

2) 5 + 8 + 11 + …………….. Find S_{10} using suitable formula

3) 5 + 10 + 15 + …….…….. Find S_{20} using suitable formula

4) 3, 7, 11, 15,……………..... Find S_{16} using suitable formula

5) 10, 15, 20,……………….. Find S_{20} using suitable formula

6) If $a_n = 2n + 1$ find the common difference ?

III. Application questions : (3/4 Marks)

1. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.
2. The 7th term of an AP is 4 times its 2nd term and 12th term is 2 more than 3 times of its 4th term. Find the progression.
3. The sum of five terms of the A.P is 55 and fourth term is 5 more than sum of first two terms of A.P. Find the terms of the A.P.
4. In an A.P sixth term is one more than twice the third term. The sum of the fourth and fifth terms is five times the second term of the A.P.
5. The common difference of two different A.P is equal. The first term of the first progression is 3 more than the first term of second progression. If the 7th term of first progression is 28 and 8th term of second progression is 29, then find the both different A.P.

6. The sum of first 9 terms of the A.P is 144 and 9th term is 28. Find the first term and common difference of the A.P.
7. The sum of 2nd and 4th terms of an AP is 54 and the sum of its first 11 terms is 693. Find the arithmetic progression. Which term of this progression is 132 more than its 54th term?

3. LINEAR EQUATIONS IN TWO VARIABLES

General form: $a_1x+b_1y+c_1 = 0$

$a_2x+b_2y+c_2 = 0$

Ratio	Graphical representation	Algebraic interpretation
$\frac{a1}{a2} \neq \frac{b1}{b2}$	Intersecting lines	Exactly one solution (unique)
$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$	Coincident lines	Infinitely many solution
$\boldsymbol{\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}}$	Parallel lines	No solution

Solve the following linear equation by elimination method:

1. 2x+y=10
   3x-y=5
   Solution:
   2x + y =10 ------→ (1)
   3x – y = 5 -----→ (2)
   5x = 15 adding 1 and 2
   $x = \frac{15}{5}$
   x = 3
   Substituting the value of x in equation (1)
   2x + y= 10
   2(3) + y = 10
   6+y = 10
   y= 10-6
   y=4
   x = 3 and y = 4

For practice :

Solve the following pair of linear equations:

1. x+ y = 14 and x-y=4
2. 2x + y = 11 and x+y = 8
3. 2x+3y = 11 and 2x -4y = -24
4. 3x + 2y = 11 and 5x -2y = 13
5. 2x + 3y = 7 and 2x + y = 5
6. 3x +y = 12 and x +y = 6
7. 2x + y = 8 and x-y = 1
8. 2x + 3y = 7 and 2x + y =5

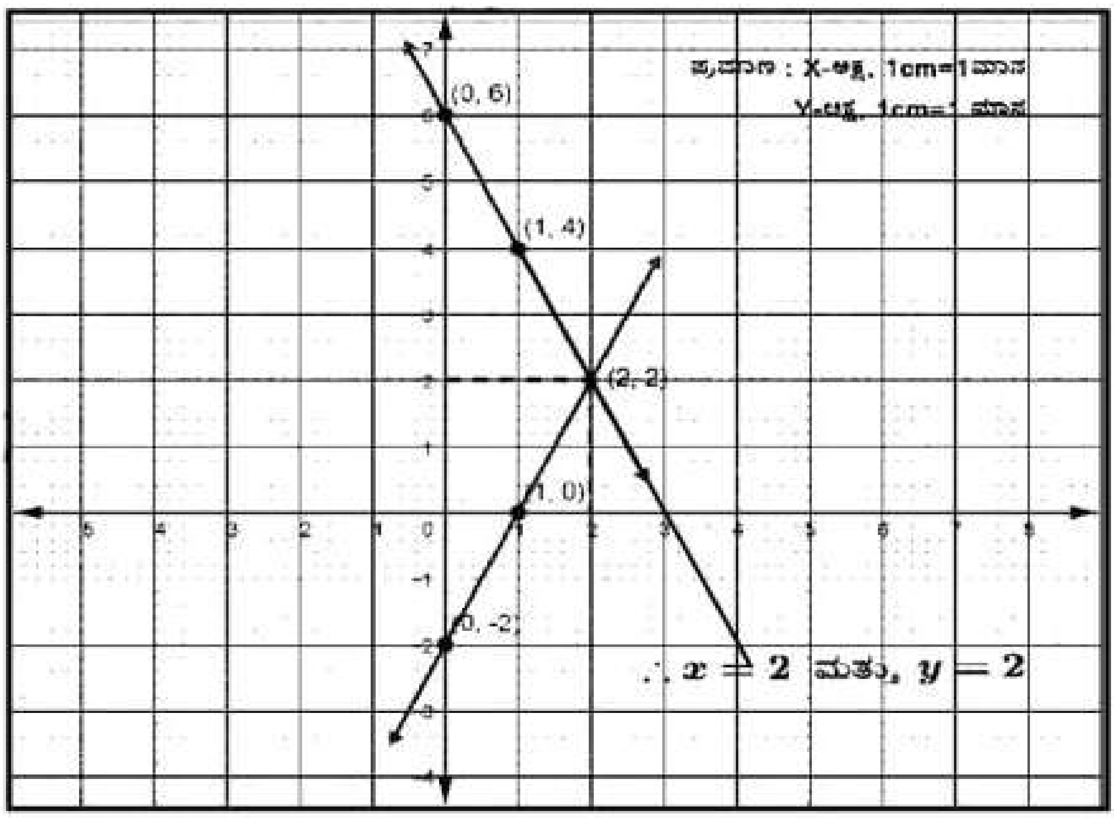
Solve the linear equation by graphical method

1. 2x+y = 6 and 2x-y = 2

   y = 6-2x and Y= 2x-2

X	0	1	3
Y	6	4	0

X	0	1
Y	-2	0



For practice

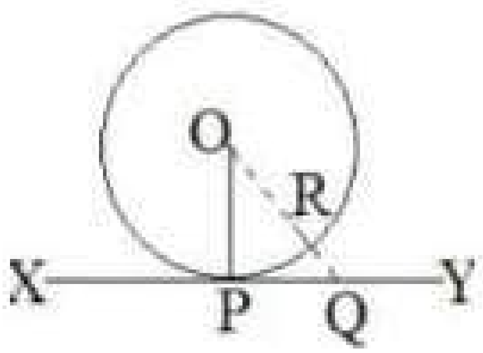
Solve the linear equation by graphical method

1. 2x+y=6 and 4x-2y=4
2. x+y=10 and x-y = 4
3. 2x-2y -2 =0 and 2x +y-6 = 0
4. 2x+y=8 and x-y=1
5. 2x+3y=12 and x-y=1
6. X+y=7 and 3x-y=1
7. 2x+y=8 and x+y =5
8. 2x+y=10 and x+y=6
9. x+y=5 and 2x-y=4
10. x+2y=6 and x+y=5
11. 2x-y =7 and x-y=2

4.CIRCLES

THEOREM

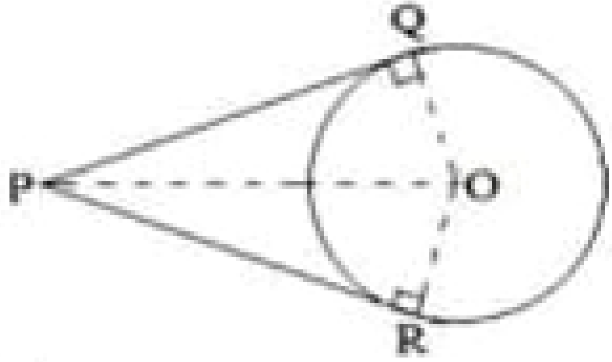
Statement: The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Data: 'O' is the centre of the circle, XY is the tangent at 'P', OP is the radius of the circle.
To Prove: OP⊥ XY
Construction: Mark a point Q on XY other than P, join OQ
let OQ intersect the circle at R
Proof: OR< OQ
But OP=OR (radii of the same circle)
Therefore OP<OQ
Since Q is the point other than P on XY, OP is the shortest of all the distances from O to XY
Therefore OP⊥ XY

THEOREM

Statement : Prove that the lengths of tangents drawn from an external point to a circle are equal



Data: 'O' is the centre of the circle. P is the external point, PQ, PR are the tangents from an external point P

To prove : PQ=PR

Construction : Join OQ, OR and OP

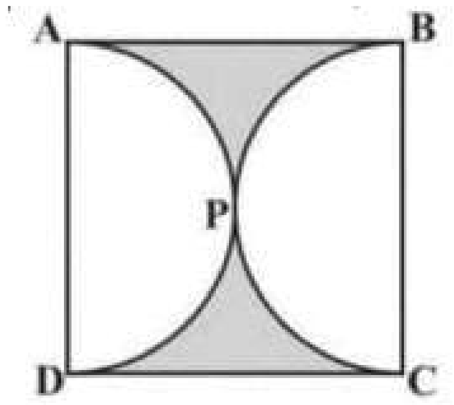
Proof:

Statement	Reasons
In triangle OPQ and triangle OPR $\llcorner OQP = \llcorner ORP = 90°$	Tangents drawn at the point of contact is perpendicular
OP=OP	Common side
OQ=OR	Radii of the same circle
$\Delta OQP \cong \Delta ORP$	RHS postulate
PQ=PR	CPCT

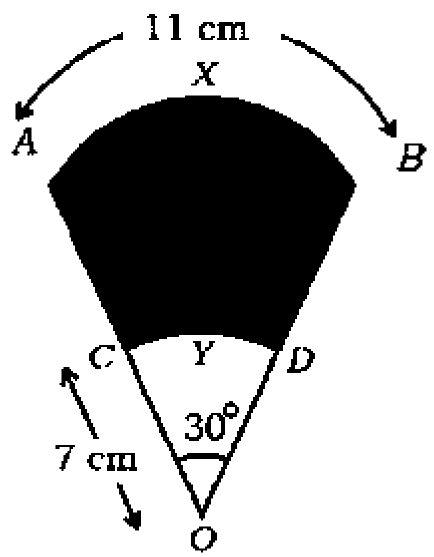
5. AREAS RELATED TO CIRCLES

- Circumference of the circle C = $2\pi r$
- Area of the circle A = πr^2
- Length of the arc of a sector of angle $\theta = \frac{\theta}{360} \times 2\pi r$
- Area of the sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$

1. In the fig ABCD is a square two semicircles APD and BPC touch each other externally at P . Length of each semi circles is 11cm . Find the area of the shaded region.



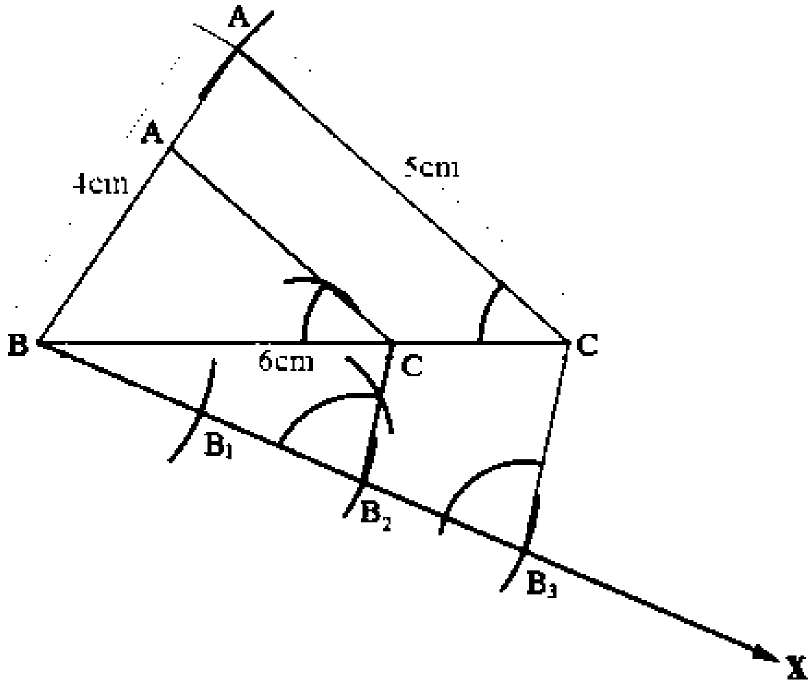
2. In the fig AXB and CYD are the arcs of two concentric circles. If length of the arc AXB is 11cm and OC=7cm and $\angle AOB = 30°$, find the area of the shaded region (take π 22/7)



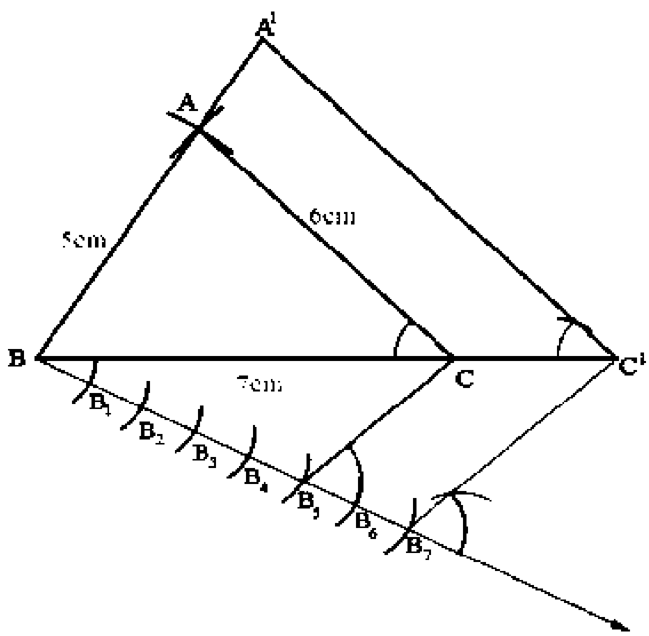
6. CONSTRUCTIONS

I Construct a triangle similar to a given triangle.

1. Construct a triangle of sides 4cm, 5cm, and 6cm and then a triangle similar to a whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.



2. Construct a triangle with sides 5cm, 6cm and 7cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.



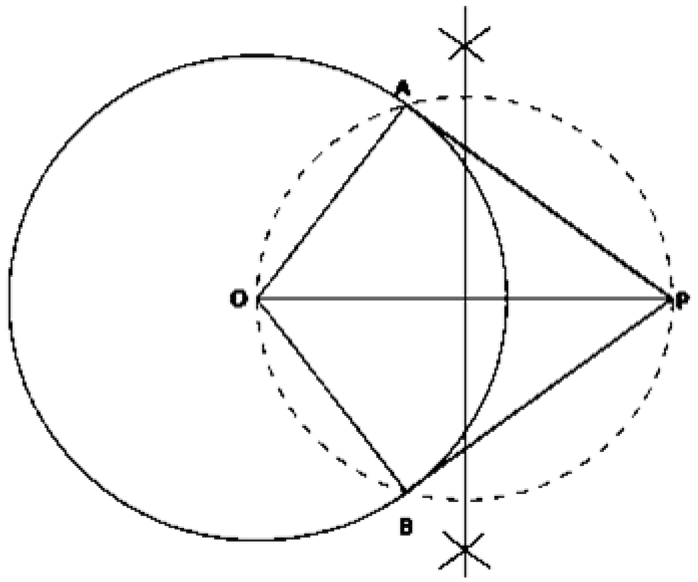
Exersise:

1. Construct a triangle with sides 5cm, 6cm,and 8cm and then another triangle whose sides are ¾ of the corresponding sides of the first triangle.
2. Construct a triangle with sides 5cm, 6cm and 7cm and then another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the first triangle.
3. Draw a triangle ABC with sides BC = 3cm, AB=6cm and AC=4.5cm, then construct a triangle whose sides are $\frac{4}{3}$of the corresponding sides of the triangle.
4. Construct a triangle of sides 6cm, 7cm, and 8cm and other triangle similar to a whose sides are ¾ of the corresponding sides of the first triangle.
5. Construct a triangle of sides 4cm, 5cm and 8cm and then a triangle similar to it whose sides are $\frac{5}{3}$ of the corresponding sides of the first triangle.
6. Construct a triangle of sides 4.5cm, 6cm and 8cm and then construct a triangle similar to it, whose sides are $\frac{5}{3}$ of the corresponding sides of the first triangle.
7. Draw a triangle ABC with sides BC = 6cm, AB = 5cm and AC = 4.5cm then construct a triangle whose sides are $\frac{4}{3}$of the corresponding sides of the first triangle.

CONSTRUCTION OF TANGNETS FROM AN EXTERNAL POINT TO A CIRCLE.

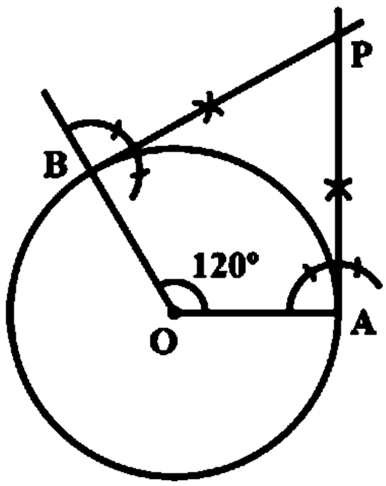
1. Draw a circle of radius 6cm from a point 10cm away from its centre. Construct pair of tangents to the circle and measure their length.

Date: d = 10cm, r = 6cm



Exercise:

1. Draw a pair of tangents to a circle of radius 4cm, such that the angle between the tangents is 60^0.

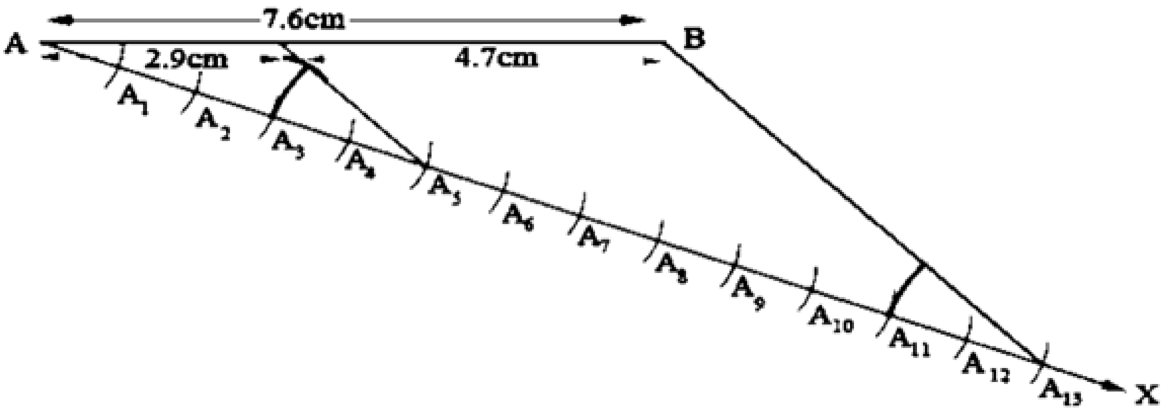


2. Draw a pair of tangents to a circle 3.5cm which are inclined to each other at an angle of 80^0.
3. Draw a pair of tangents to a cicle of radius 3cm such that the angle between the tangent is 60^0.
4. Draw a pair of tangents to a circle of radius 4cm such that the angle between the tangents is 70^0.

5. Draw a circle of radius 3.5cm and draw two radii such that the angle between the two tangents is 80^0 and draw tangents to the circle at their end points.
6. Draw two tangents from a point 8cm from the centre to a circle of radius 3cm.

TO DIVIDE A LINE SEGMENT IN THE GIVEN RATIO

1. Draw a line segment of length 7.6cm and divide in the ration 5:8.
   Data: Length of segment 7.6cm and ratio 5:8



Exercise:
1. Draw a line segment of length AB = 8cm and divide in the ratio 3:2
2. Draw a line segment of length 10cm and divide in the ratio 2:3
3. Draw a line segment of length 8.4 cm and divide in the ratio 1:3

7.COORDINATE GEOMETRY

- Distance between two coordinates $d=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Distance between a point (x, y) from the origin : $d = \sqrt{x^2 + y^2}$
- Section formula (x, y) = $(x, y) = [\frac{m_1x_2+m_2 x_1}{m_1+m_2}, \frac{m_1y_2+m_2 yx_1}{m_1+m_2}]$
- Mid point formula : $(x, y) = [\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}]$
- Area of triangle formula : $\frac{1}{2} [x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Distance between two coordinates

1. Find the distance between (2,3) and (4,1).
   Solution: Let A(2,3) and B(4,1)

$\boldsymbol{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(4 - 2)^2 + (1 - 3)^2}$

$AB = \sqrt{(2)^2 + (-2)^2}$

$AB = \sqrt{4 + 4}$

$AB = \sqrt{2 \times 4}$

$AB = 2\sqrt{2}$ units

Exercise: 1. Find the distance between (2,3) and (4,1).

2. Find the distance between (-5,7) and (-1,3).
3. Find the co-ordinates of the point on the line segment joining the points (2,3) and (10,-3).
4. Find the distance between the points (3,6) and (5,7) using distance formula.
5. Find the distance between the points (2,6) and (9,10) using distance formula.
6. Find the distance between the points (2,3) and (4,1) using distance formula.
7. Find the distance of a point (3,4) from the origin.
8. Find the distance between (7,-8) and (-3,0).

2 or 3 Marks

1. A circle whose centre is at p(2,3) passes through the points A(4,3) and B(x,5) then find the value of x.

The points (4,3) and (x,5) are the same distance from the point (2,3)

AP = BP

$\sqrt{(4 - 2)^2 + (3 - 3)^2} = \sqrt{(2 - x)^2 + (3 - 5)^2}$

$4 + 0 = (2-x)^2 + 4$

$4 - 4 = (2-x)^2$

$0 = 2 - x$

$x = 2$

2. Show that (8,-4), (9,5) and (0,4) are the vertices of an isosceles Triangle.
   Let A (x_1, y_1) = (8,-4), B (x_2, y_2) = (9,5) and C (x_3, y_3)=(0,4)
   From the distance formula

$AB=\sqrt{(9-8)^2 + (5+4)^2}$

$=\sqrt{(1)^2 + (9)^2}$

$=\sqrt{82}$

$BC=\sqrt{(0-9)^2 + (4-5)^2}$

$=\sqrt{81+1}$

$=\sqrt{82}$

AB = BC

ABC is an isosceles triangle.

Exercise:

1. Find the co-ordinates of the point which divides the line joining (4,-3) and (8,5) in the ratio 3:1.
2. Find the co-ordinates of the point AB divides internally joining the points (-1,7) and (4,-3) in the ratio 2:3.
3. (2,x) divides the line joining the points A(-2,2) and B(3,7) internally. Find the ratio in which it divides and also find the value of x.
4. Find the co-ordinates of the mid point of their line segment joining the points (2,3) and (4,7).
5. Find the area of triangle whose vertices are (1,-1), (-4,6) and (-3,-5).
6. If $(-\mathbf{3},\ \mathbf{2})$, $(-\mathbf{1},\ -\mathbf{4})$ and $\boldsymbol{C}(\mathbf{5},\ \mathbf{2})$ are the vertices of $\Delta\ \boldsymbol{ABC}$, $\boldsymbol{M}$ and $\boldsymbol{N}$ are the mid points of $\boldsymbol{AB}$ and $\boldsymbol{AC}$ respectively then show that $\mathbf{2}\boldsymbol{MN}=\boldsymbol{BC}$.
7. The points A(1,1), B(3,2) and C(5,3) cannot be the vertices of the triangle ABC. Justify

8. In each of the following find the value of 'k' for which the points are collinear. (2,-2), (-4,2) and (-7,k).
9. Find the value of 'k' for which the points are collinear (2,3), (4,k) and (6,-3).

8.REAL NUMBERS

- Euclid's division lemma : a = (b x q) + r
  Dividend = (divisor x quotient) + remainder
- HCF of (a,b) x LCM of (a,b) = a x b

I Prove that 3 + $\sqrt{5}$ is an irrational number.

Solution: Assume that 3 + $\sqrt{5}$ is a rational number

$$\therefore 3 + \sqrt{5} = \frac{p}{q} \; [p, q \in z, q \neq 0]$$

$$\sqrt{5} = \frac{p}{q} - 3$$

$\frac{p-3q}{q}$ is a rational number,but $\sqrt{5}$ is an irrational number,

So our assumption is wrong.

$\therefore$ 3 + $\sqrt{5}$ is an irrational number

Exercise:

1. Prove that 5 - $\sqrt{3}$ is an irrational number.
2. Prove that 3 + $\sqrt{2}$ is an irrational number.
3. Prove that 5 + $\sqrt{3}$ is an irrational number.
4. Find the HCF of 24 and 40 by using Euclidis division algorithm. Hence find the LCM of HCF (24,40) and 20.
5. Given that HCF of (306, 657) is 12. Find the LCM.

9. POLYNOMIALS

General Form

1) Linear polynomial p(x) = ax + b
2) Quadratic polynomial p(x) = $ax^2 + bx + c$
3) Cubic polynomial p(x)= $ax^3 + bx^2 + cx + d$

GENERAL FORM OF POLYNOMIAL

$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; a_nx^n$

If α and β are the zeros of the polynomial $ax^2 + bx + c$ then,

Sum of zeros $\alpha + \beta = \frac{-b}{a}$ or $\alpha + \beta = \frac{-(co-efficient\ \ of\ x)}{co-efficient\ \ of\ x^2}$

Product of zerox $\alpha\beta = \frac{c}{a}$ or $\alpha\beta = \frac{constant\ \ term}{co-efficient\ \ of\ x^2}$

If $\alpha, \beta and\ \gamma$ the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

Sum of zeros $\alpha + \beta + \gamma = \frac{-b}{a}$ OR $\frac{-(co-efficient\ \ of\ x)}{co-efficient\ \ of\ x2}$

Product of zeros $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ OR $= \frac{constant\ \ term}{co-efficient\ \ of\ x2}$

DIVIDEND = DIVISION X QUOTIENT + REMINDER

p(x) = {g(x) x q(x)}+ r(x)

Find the maximum number of zeroes in the following

1) Linear polynomial

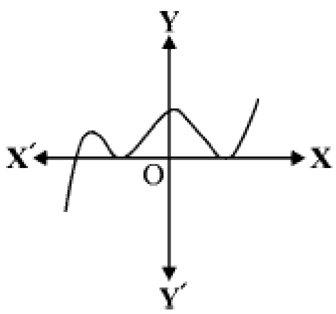
2) Quadratic polynomial

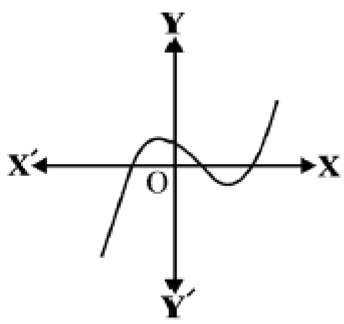
3) Cubic polynomial of

4) The degree of P(x)=**x^5+3 x^3 – 7x^2-9x+11.** is

a) 1 b)2 c)3 d) 5

5) Find the number of zeroes of the polynomial represented in the graph.





6). Find the zeros of the Quadratic Polynomial $p(x)=x^2 + 7x + 10$ and verify the relationship between zeros and the co-efficient.

Solution: $P(x)= x^2 + 7x + 10$

$0= x^2 + 5x + 2x + 10$

$0= x(x+5) + 2(x+5)$

$x+5 = 0$ or $x+2 = 0$

$x = -5$ or $x= -2$

Sum of zeros = (-5) + (-2) = (-7)

$$\frac{-7}{1} = \frac{co-efficient\ \ of\ x}{co-efficient\ \ of\ x2}$$

Product of zerox = (-5) x (-2) = 10

$$\frac{10}{1} = \frac{constant\ \ term}{co-efficient\ \ of\ x^2}$$

7).Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$ and verify the division algorithm

$-x^2 + x - 1)\ -x^3 + 3x^2 - 3x + 5\ (\ x - 2$

$- x^3 + x^2 - x$

$(+)\ \ (-)\ \ (+)$

$2x^2 - 2x + 5$

$2x^2 - 2x + 2$

$(-)\ \ (+)\ \ (-)$

3

Divisor x Quotient + Remainder

$= (-x^2 + x - 1)\ (x - 2\ ) + 3$

$= -x^3 +x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 -3x + 5$

= dividend

For Practice

1) If one zero of the polynomial $p(x) = x^2 -6x + k$ is twice the other. Find the value of k.
2) Find the polynomial of least degree that should be subtracted from $p(x) = x^3 - 2x^2 + 3x + 4$. So that it is exactly divisible by $g(x) = x^2 - 3x + 1$.
3) If the sum of the zeros of the polynomial $p(x) = 2x^2 - 6x + k$ is half of the product of the zero. Find k.

4) If α and β are the two zeros polynomial $p(x) = 3x^2 - 12x + 15$.
Find $\alpha^2 + \beta^2$

5) Write the quadratic polynomial whose sum and product are respectively ¼ and -1

6) The sum and product of the zeros of a quadratic polynomial $p(x) = ax^2 + bx - 4$ are ¼ and -1 respectively. Then find the value of a and b.

7) If $p(x) = 3x^3 + x^2 + 2x + 5$ is divisible by g(x), then the quotient and remainder are respectively. $(x - 2)$ and $(-2x + 4)$. Find g(x).

8) Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$ and verify the division algorithm.

10. QUADRATIC EQUATION

Standard Form of a quadratic equation

$ax^2 + bx + c = 0$ wh a,b,c are real numbers and $a \neq 0$

Methods to solve quadratic equation

1) Factorization method
2) Completing the square
3) Using formula method

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Descriminant of $ax^2 + bx + c = 0$ is $b^2 - 4ac$ (Δ)

1) If $b^2 - 4ac > 0$ Q.E. has two distinct real roots.
2) If $b^2 - 4ac = 0$ Q.E. has two equal real roots
3) If $b^2 - 4ac < 0$ Q.E. has no real roots

1. Find the roots of the quadratic equation $6x^2 - x - 2 = 0$, by factorization.

$6x^2 - x - 2 = 0$

$6x^2 + 3x - 4x - 2 = 0$

$3x(2x+1) - 2(2x+1) = 0$

$(3x-2)(2x+1) = 0$

$3x-2 = 0, \quad 2x+1=0$

$3x = 2, \quad 2x = -1$

$x = \frac{2}{3}, \quad x = \frac{-1}{2}$

2. Solve the quadratic equation $2x^2 + x - 528 = 0$ by using formula.

$2x^2 + x - 528 = 0$

$ax^2 + bx + c = 0$

a = 2, b = 1, c = -528

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1^2 - 4(2)(-528)}}{2(2)}$

$= \frac{-1 \pm \sqrt{1 + 4224}}{4}$

$= \frac{-1 \pm \sqrt{4225}}{4}$

$= \frac{-1 \pm 65}{4}$

$= \frac{-1+65}{4}$, or $x = \frac{-1-65}{4}$

$= \frac{64}{4}$ or $x = \frac{-66}{4}$

$= 16$ or $x = \frac{-33}{2}$

3. Find the discriment of the quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots.

$3x^2 - 2x + 1/3 = 0$

$ax^2 + bx + c = 0$
a = 3, b = -2, c = 1/3

discriment $b^2 - 4ac = (-2)2 - 4(3)(1/3)$
$= 4 - 4$
$= 0$
$b^2 - 4ac = 0$
∴ The given equation has two equal real roots.

Exercise:
1) Solve $x^2 + 5x + 6 = 0$ by factorization

2) Find the roots of the quadratic equation $x^2 - 2x + 3 = 0$ by applying the quadratic formula.
3) Find the discriminate of the quadratic equation $2x^2 - 5x + 3 = 0$ and hence find the nature of the roots.
4) The length of the rectangular field is three times its breadth. The area of the field is 147 square mt. Find the length and breadth of the rectangular field
5) Sum of two natural numbers is 9. The sum of their reciprocal is 9/20. Find the numbers.
6) Let the two digit number is four times the sum of its digit and three times the product of its digits. Find the number.

13. STATASTICS

MEAN	Mean : $\sum \frac{fx}{n}$ OR $\sum \frac{f_i x_i}{f_i}$
MODE	Mode = $L + \left[\frac{f1-f0}{2f1-f0-f2}\right] \times h$
MEDIAN	Median = $L + [\frac{n/2-cf}{f}] \times h$

I Find the mean for the following data:

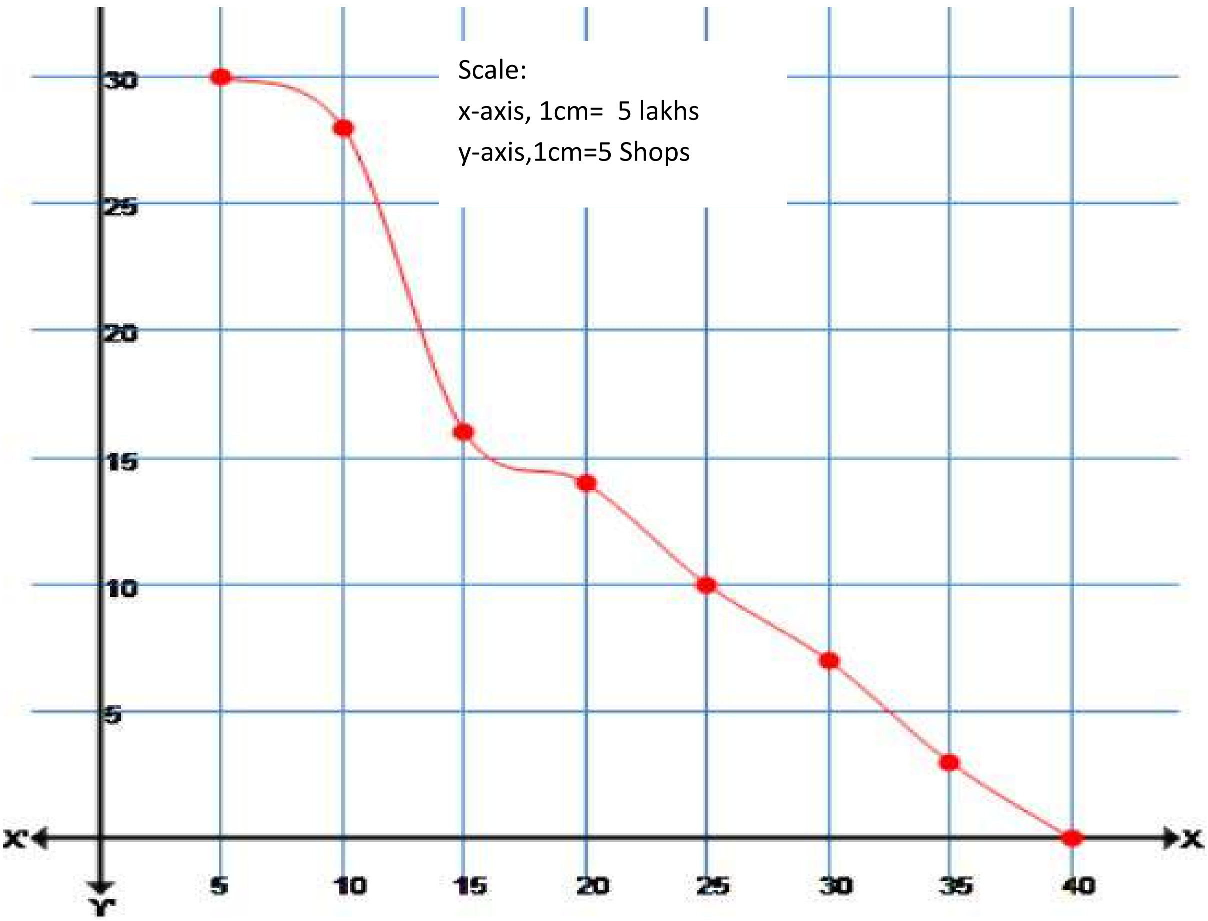
Class Interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 - 100
Frequency	**2**	**3**	**7**	**6**	**6**	**6**

Class Interval	Frequency (f)	Midpoint (x)	fx
10 – 25	2	17.5	35.0
25 – 40	3	32.5	97.5
40 – 55	7	47.5	332.5
55 – 70	6	62.5	375.0
70 – 85	6	77.5	465.0
85 - 100	6	92.5	555.0
Total	$\sum f = 30$		$\sum fx = 1860$

Mean $\bar{x} = \sum \frac{fx}{n} = \frac{1860}{30} = 62.$

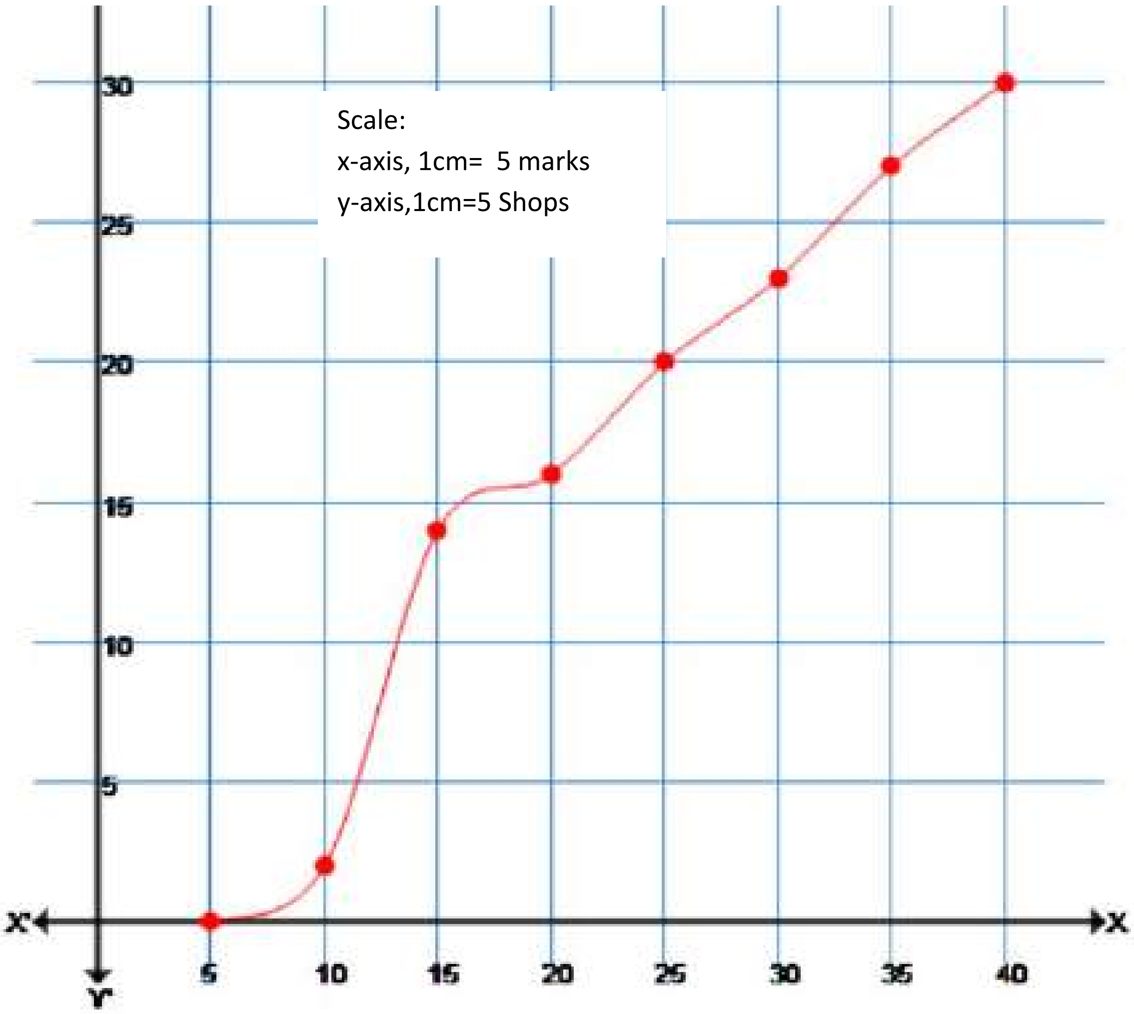
1). Annual profit carried by 30 shops of a complex in a locality give rise to the following distribution. Draw more than type Ogive.

Profit (Rs in Lakhs)	Number of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3



1. The details of marks obtained by students in a unit test held in a school are as follows. Draw less than type Ogive.

Marks	No. of students (cumulative frequency)
Less than 10	2
Less than 15	14
Less than 20	16
Less than 25	20
Less than 30	23
Less than 35	27
Less than 40	30



Exercise :

`1. Find the Mean for the following data:

Class Interval	Frequency
0 – 10	10
10 – 20	18
20 – 30	8
30 – 40	9
40 - 50	5

2. Marks of 10^{th} class students are given below. Find the Mode.

Marks	Frequency
0 – 20	6
20 – 40	12
40 – 60	10
60 – 80	9
80 - 100	7

3. Find the Median for the following data:

Marks	Frequency
1 – 4	6
4 - 7	30
7 - 10	40
10 - 13	16
13 – 16	4
16 - 19	4

4. During the Medical check up of 60 students of a class their weights were recorded as follows: Draw more than type Ogive.

Weight (in Kg)	No. of students
More than or equal to 38	60
More than or equal to 40	48
More than or equal to 42	25
More than or equal to 44	18
More than or equal to 46	10
More than or equal to 48	5

5. During the medical check up of 45 students of a class, their weights were recoreded as follows: Draw less than type Ogive.

Weight (in Kg)	No. of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	25
Less than 50	35
Less than 52	45

14. PROBABALITY

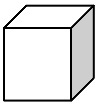
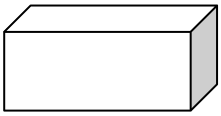
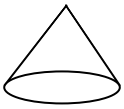
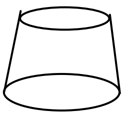
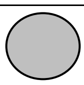
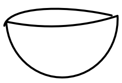
Probability of an even	P(A) = $\frac{n(A)}{n(S)}$
Probability of Sure event	1
Probability of impossible event	0
Formula for Probability of event and its complementary event	p(A) + p($\bar{A}$) = 1
Random experiment with single coin tossed	(S) = {H,T}, n(S) = 2
Random experiment with two coin tossed	(S) = {HH, TT, HT, TH}, n(S) = 4
Random experiment with three coin tossed	(S) = {HHH, TTT, HHT, TTH, HTH, THT, THH, HTT}, n(S) = 8
Possible outcome of dice	(S) = {1, 2, 3, 4, 5 , 6}, n(S) = 6
Possible outcomes when two dice of six faces thrown at a time	(S) = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)} {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)} {(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)} {(4,1), (4,2),,(4,3), (4,4), (4,5), (4,6)} {(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)} {(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)} n(S) = 36

Exercise:

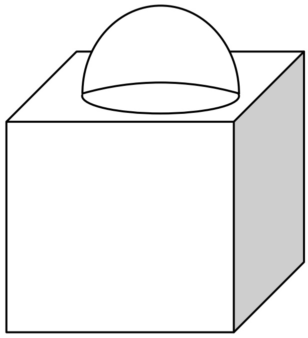
1. A bag contains 3 red balls, 5 white balls and 8 blue balls. A ball is drawn at random from the tag. What is the probability of getting,
   a) A red ball , b)Not a white ball.
2. A box contains 80 discs which are number from 1 to 80. If one disc is drawn at random from the box. What is the probability of getting a square number.

3. A game consist of tossing 1Rs coin 2 times and nothing its outcome each time. Raghu wins if all the tosses give same result. Two heads or two tails and looses otherwise. Calculate the probability that Raghu will lose the game..
4. A Die is thrown once. What is the probability of getting,
   a) A prime number. b) A number between 2 and 6.
5. Two players Sangeetha and Rekha play a tennis match. It is known that the probability of Sangeetha winning the match is 0.62 what is the probability that Rekha will win the match?
6. There are 4 Apples,6 Orenges and some Gouva fruits in a box a fruit is drawn randmly from the box the probability of drawing an Orenge is $\frac{6}{13}$ Find the number of gouva fruit in the box.

15. SURFACE AREA AND VOLUME

SOLID	(Lateral Surface Area-L.S.A)	(Total Surface Area-T.S.A)	Volume
CUBE 	$A = 4a^2$	$A = 6a^2$	$V = a^3$
CUBOID 	$A = 2h(l + b)$	$A = 2(lb + bh + hl)$	$V = l\ b\ h$
CONE 	$A = \pi\ rl$	$A = \pi r(r + l)$	$V = \frac{1}{3}\pi r^2 h$
FRUSTRUM 	$A = \pi(r_1 + r_2)l$	$A = \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$	$V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2\}$
SPHERE 	$A = 4\pi r^2$	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
HEMISPHERE 	$A = 2\pi r^2$	$A = 3\pi r^2$	$V = \frac{2}{3}\pi r^3$

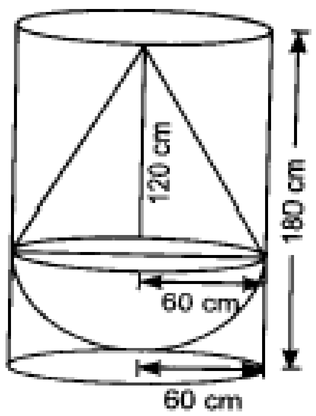
1. A show piece is made up of a cube and semisphere the edge of the cube is 5cm in length and the hemisphere with diameter 4.2cm is at the top of the cube. Find the total surface area [$\pi = \frac{22}{7}$]



Area of cube = 6 x (side)^2
= 6 x 5^2
150 cm^2.
TSA = [TSA of cube – base of hemisphere + LSA of hemisphere]
= [156 - πr^2 + 2πr^2]
= [150 + πr^2]
= [150 + 13.86]
= 163.86 cm^2

Exercise:

1. Find the volume and surface area of a frustrumits with base radii 15cm and 8cm and its depth 24 cm.
2. A cone with radius 5cm and height 20cm is melted and recast into the shape of a sphere. Find the radius of the sphere.
3. The surface area of acone and a cylinder are same. The radius and height of the cylinder are 7cm and 10 cm respectively. If the slant height of the cone is 14 cm then find the radius of the cone.
4. Find the total surface area and volume of a cuboid of dimissions 10cm x 6cm x 8cm
5. A solid made up of a cone and a hemisphere is immerced in a cylndrical container with full of water Find the quantity of water retained in the cylinder



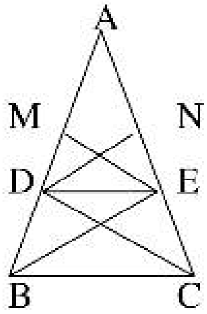
2. Triangles

1. Thales theorem or basic proportional theorem.

Statement :- If a line is drawn parallel to one side of triangle then it intersect other two sides in same ratio.

we have $\triangle$ ABC

where DE//BC



to prove

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction :- Draw on DN $\perp$ AC

And EM $\perp$ AB also join DC and BE

$$\frac{Area\ of\ \Delta\ ADE}{Area\ of\ \Delta\ BDE} = \frac{\frac{1}{2}X\ AD\ X\ EM}{\frac{1}{2}X\ BD\ X\ EM} = \frac{AD}{BD} \quad -(i)$$

Also $\frac{Ar\Delta\ ADE}{Ar\ \Delta\ CED} = \frac{\frac{1}{2}X\ AE\ X\ DN}{\frac{1}{2}X\ EC\ X\ DN} = \frac{AE}{EC} \quad -(ii)$

Since we know that triangle between same parallel and having same base are equal in area.

So, Ar of Δ BDE = Ar Δ CED

$$\frac{1}{Ar\Delta\ BDE} = \frac{1}{Ar\Delta\ CED}$$

Both side multiply by Ar Δ ADE

$$\frac{Ar\Delta\ ADE}{Ar\Delta BDE} = \frac{Ar\Delta\ ADE}{Ar\Delta\ CED}$$

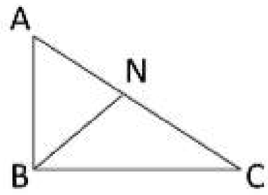
from (i) & (ii)

$$\frac{AD}{BD} = \frac{AE}{EC}$$

Hence proved the Thales theorem.

2. State and prove Pythagoras theorem.

Statement: In a right angle triangle
Square of largest side
Is equal to the sum of the
Square of other two sides.



We have triangle ABC where <B = 90°

To prove $AC^2 = AB^2 + BC^2$

Construction : Drawn $BN \perp AC$

In Δ ABC and Δ ABN

<ABC = < BNA each 90°

<A = <A common

then from A.A. similarity

Δ ABC ~ Δ ABN

$\frac{AB}{AN} = \frac{AC}{AB}$

$AB^2 = AN \ X \ AC \ - \ (i)$

Again in Δ ABC and Δ BNC

<ABC = <BNC = 90°

<C = <C common

then from A.A similarity

Δ ABC ~ Δ BNC

$\frac{BC}{NC} = \frac{AC}{BC}$

$BC^2 = NC \ X \ AC \ - (ii)$

Adding (i) & (ii)

$AB^2 + BC^2 = AN \ X \ AC + NC \ X \ AC$

$AB^2 + BC^2 = AC (AN + NC)$

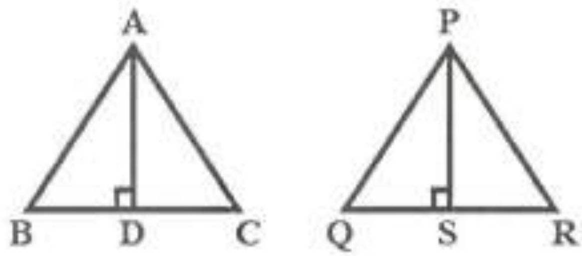
$AB^2 + BC^2 = AC \ X \ AC$

$AB^2 + BC^2 = AC^2$ Proved

3. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Proof : We are given two triangles ABC and PQR such that

To prove : Ratio of areas of ΔABC and ΔPQR is equal to the square of the ratio of their corresponding sides.



Given : $\Delta ABC \sim \Delta PQR \ldots (1)$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \ldots (2)$$

$\angle B = \angle Q \ldots (3)$

In ΔABC and ΔPQS

$\angle B = \angle Q$ [from (3)]

$\angle ADB = \angle PSQ = 90^\circ$

$\therefore \Delta ABD \sim \Delta PQS$ [By AA similarity]

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PS} \ldots (4)$$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$$

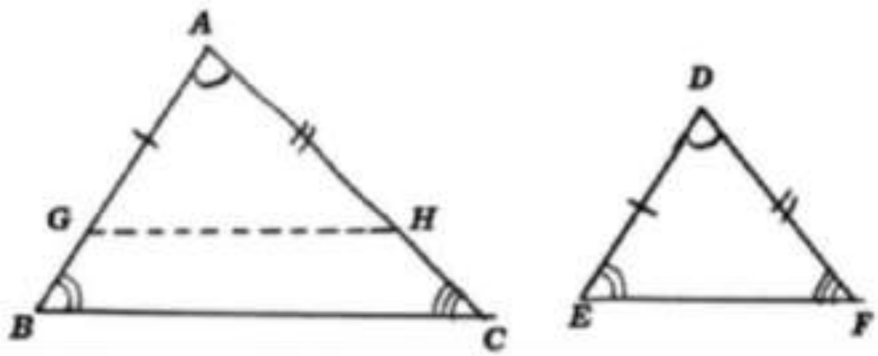
$$= \frac{BC}{QR} \times \frac{AD}{PS} = \frac{AB}{PQ} \times \frac{AB}{PQ} \text{ [From (2) \& (4)]}$$

$$= \left(\frac{AB}{PQ}\right)^2$$

Hence Proved

4. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar. This criterion is referred to as the AAA (Angle–Angle–Angle) criterion of similarity of two triangles.

Figure:

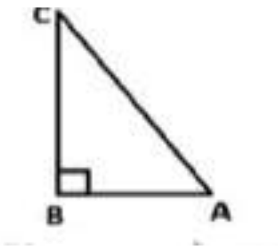


data:	ΔABC and ΔDEF $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$
To prove:	$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
Construction:	Draw G and H on AB and AB such that AG = DE and AH =DE. Join G and H.

Proof: Statement	**reasons**
In ΔABC and ΔDEF,	
$\angle A = \angle D$	{given}
AG=DE, $\angle A = \angle D$	{construction}
ΔABC ≅ ΔDEF	{SAS Criteria}
Therefore ∠G = ∠E	{by CPTC}
But, $\angle B = \angle E$	
and Thus ∠G = ∠E	
∴ BC ∥ GH	
From ΔABC $\frac{AB}{AG} = \frac{BC}{GH} = \frac{AC}{HA}$	From Thales converse
$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{FD}$	ΔABC ≅ ΔDEF

11. Introduction toTrigonometry

(1) Ratios



- Sin (A) = $\frac{BC}{AC}$ * cosecant (A) = $\frac{AC}{BC}$
- Cos (A) = $\frac{AB}{AC}$ * sec (A) = $\frac{AC}{BC}$
- tan (A) = $\frac{BC}{AB}$ * cot (A) = $\frac{AB}{BC}$

(b)Standard Angles

θ	0^0	30^0	45^0	60^0	90^0
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
cosec A	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D
cot A	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

1. Ratio of complementary angles

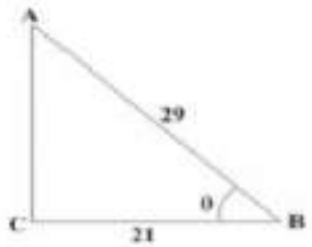
   Sin (90°.A) = cos   * cosec (90°.A) = sec A

   Cos (90°.A) = sin A   * sec (90°.A) = cosec A

   tan (90°.A) = cot A   * cot (90°.A) = tan A

2. Regular equation * $\sin^2 A + \cos^2 A=1$
   - $1+ \tan^2 A=\sec^2 A$
   - $\cot^2 A +1 = \text{cosec}^2 A$

1. If in ΔACB, AB =29 units. BC = 21 units and ABC = θ. Find the Value of (i) $\cos^2\theta + \sin^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$



In Δ ACB

AC= $\sqrt{AB^2 - AC^2}$

$= \sqrt{(29)^2 - (21)^2}$

$= \sqrt{(29-21)-(29+21)} = \sqrt{8(50)} = \sqrt{400} = 20\ units$

Therefore $\sin\theta = \frac{AC}{AB} = \frac{20}{29}, \cos\theta = \frac{BC}{AB} = \frac{21}{29}$

Now (i) $cos^2\theta + \sin 2\theta = \left(\frac{20}{29}\right) + \left(\frac{21}{29}\right) = 20^2 + 21^2 = \frac{400+441}{29^2} = \frac{841}{841} = 1$

And (ii) $cos^2\theta + \sin 2\theta = \left(\frac{21}{29}\right) + \left(\frac{20}{29}\right) = \frac{(20+20)(21-20)}{29^2} = \frac{41}{841}$

2. If sin (A-B) = $\frac{1}{2}$. cos (A+B) = $\frac{1}{2}$. 0°, < A+B ≤ 90°. A> B and find A and B

Sin (A – B) = ½ therefore A – B = 30° ___________(1)

Cos (A + B) = ½ therefore A + B 60° ___________(2)

Combining (1) and (2) ∴ A = 45° and B = 15°

3. Express cot 85°+ cot 75°. in trigonometric ratios between 0° and 45°

cot 85° + cot 75°

= cot (90° - 5°) + cot (90° - 15°)

= tan 5° + tan 15°

4. Prove that sec A (1 sin A)(sec A + tan A) = 1

LHS sec A (1 sin A)(sec A + tan A)

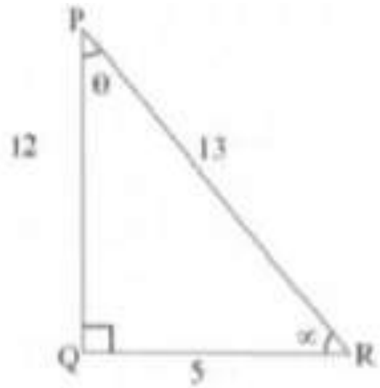
$= \left(\frac{1}{cosA}\right)(1 \sin A)\left(\frac{1}{cosA} + \frac{sinA}{cosA}\right)$

$= \frac{(1-\sin A)(1+\sin A)}{\cos^2 A} = \frac{1-\sin^2 A}{\cos^2 A}$

$= \frac{\cos^2 A}{\cos^2 A} = 1 =$ RHS

For practice

1. Find the value of $\sin\theta$, $\cos\theta$, $\tan\theta$, and $\sin \propto \cos \propto . \tan \propto$



2. Prove that cosec A (1coser A) (cosec A+ cot A) =1
3. Find the value of $\cos\theta$ and $\tan\theta$ if $\sin\theta = \frac{12}{13}$
4. Express the ratios of cos A, tan A and sec A in the form of sin A
5. Prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$
6. Find the value of sin 60°. Cos30°+sin 30° cos 60°.
7. Prove that $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$
8. Simplify $\sin 25^{o} \cos 65^{o} + \cos 25^{o} \sin 65^{o}$
9. Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

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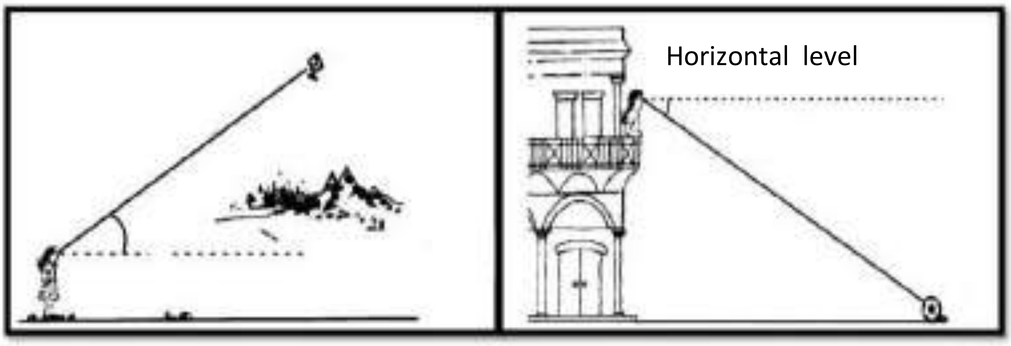
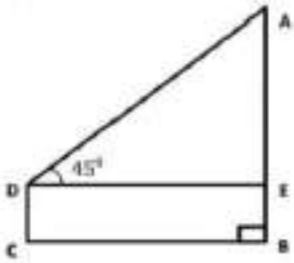
12. Some application of trigonometry



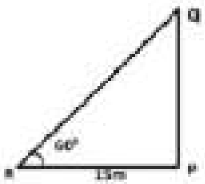
1. In figure CD= 1.5m, find the measure of AB.



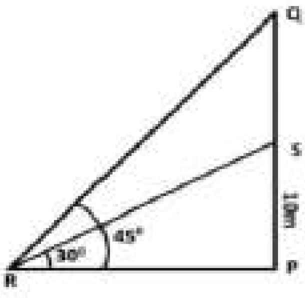
AB = AE + BE
= AE + 1.5
DE = CB = 28.5m
Tan 45º = $\frac{AE}{DE}$
1 = AE / 28. 5
∴ AE = 28.5m
AB = (28.5 + 1.5) m = 30m

For practice

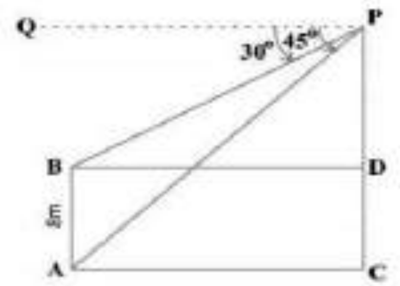
3. Find the measure of PQ in figure



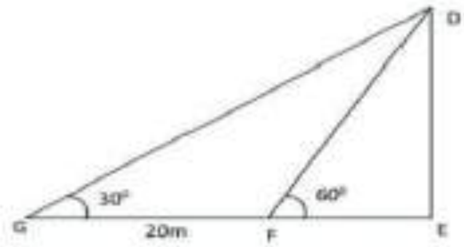
4. In the figure PS=10m, find the measure of QS and $PR(\sqrt{3} = 1.732)$



3. In the figure AB=8m, measure CP and AC



4. In the figure, GF=20m, find the measures of DE and EF.



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